

# Ex-Ante Methods to Assess the Impact of Social Insurance Policies on Labor Supply with an Application to Brazil

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## Abstract

This paper solves and estimates a stochastic model of optimal inter-temporal behavior to assess how changes in the design of the unemployment benefits and pension systems in Brazil could affect savings rates, the share of time that individuals spend outside of the formal sector, and retirement decisions. Dynamics depend on five main parameters: preferences regarding consumption and leisure, preferences regarding formal Vs. informal work, attitudes towards risks, the rate of time preference, and the distribution of an exogenous shock that affects movements in and out of the social security system (given individual decisions). The yearly household survey is used to create a pseudo panel by age-cohorts and estimate the joint distribution of model parameters based on a generalized version of the Gibbs sampler. The model does a good job in replicating the distribution of the members of a given cohort across states (in or out of the social security / active or retired). Because the parameters are

related to individual preferences or exogenous shocks, the joint distribution is unlikely to change when the social insurance system changes. Thus, the model is used to explore how alternative policy interventions could affect behaviors and through this channel benefit levels and fiscal costs. The results from various simulations provide three main insights: (i) the Brazilian SI system today might generate distortions (lower savings rates and less formal employment) that increase the costs of the system and might generate regressive redistribution; (ii) there are important interactions between the unemployment benefits and pension systems, which calls for joint policy analysis when considering reforms; and (iii) current distortions could be reduced by creating an actuarial link between contributions and benefits and then combining matching contributions and anti-poverty targeted transfers to cover individuals with limited or no savings capacity.

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This paper—a product of the Social Protection, Human Development Network—is part of a larger effort in the network to understand the interactions between the social insurance systems and the labor market.. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The author may be contacted at [drobalino@worldbank.org](mailto:drobalino@worldbank.org).

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# Ex-Ante Methods to Assess the Impact of Social Insurance Policies on Labor Supply with an Application to Brazil

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# 1 Introduction

Social insurance policies affect individual behaviors and can have non-trivial effects on the supply side of the labor market. The existence of mandatory pensions, for instance, affects retirement decisions. Often, benefit formulas that are not actuarially fair and/or minimum pension guarantees induce early withdrawals from the labor force, which increase the cost of the pension system and reduce employment levels (see, for instance, Bodor et al. 2008; Jiménez-Martin and Sánchez-Martin, 2006; Blundell et al. 2002; Anderson et al. 1999; Gruber and Wise, 1998; Samwick, 1998; Lumsdain and Wise, 1994; and Fields and Mitchel, 1988). Badly designed retirement income transfers can also reduce incentives to contribute to the social security and promote informal work (see Valdez, 2008; and Piggot et al., 2009).

Unemployment benefits schemes affect behaviors as well. Unemployment insurance systems, for example, can create moral hazard reducing incentives to search for or keep jobs. The literature usually finds a positive correlation between the level of the benefit and its duration, and the length of the unemployment spell (see, for instance, Layard et al., 2006; Card and Levine, 2000; Calmfors and Holmlund, 2000; Anderson and Mayer, 1993; and Mayer, 1991). Even funded mandatory unemployment savings accounts which are expected to be incentives neutral can, under some circumstances, have unintended consequences and induce more frequent separations and higher turnover (see Vodopivec et al., 2009).

More generally, the structure of the bundle of social security benefits and the financing mechanism determine the incentives facing individuals to take formal sector jobs (see Perry et al., 2007; and Levy, 2006). When the bundle includes benefits that are not valued by individuals or there is an important redistributive component, part of the social security contribution acts as a tax that can promote evasion and informal sector work.<sup>1</sup> At the same time, benefits that are too generous relative to contributions create implicit subsidies that can reduce incentives to work and save (i.e., self-insure).

The standard analysis of the economic impacts of policy changes in the social insurance system, however, generally ignores these effects or simply makes assumptions about possible behavioral responses (see for instance World Bank, 2006). One reason is that there are no econometric models linking the complex set of rules of a given system (say pensions) to behaviors; there is never enough variation in system parameters to estimate these models. Thus, the econometric models that we have tell us how the *presence* of a social insurance program

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<sup>1</sup> For a general review of some of the linkages between the social insurance system and the labor market see Krueger and Mayer (2002).

affects a certain behavior but not what would happen if the rules of the system change – particularly if one is interested about changes in more than one program. One could recur to pilot ex-post impact evaluations to understand how a given change in policy would affect behaviors and ultimately welfare. But these exercises are costly and not very suitable to assess *possible scenarios* for reform. In this case, we argue, a second best is to rely on a behavioral model derived from first principles and with the joint distribution of parameters constructed to maximize the likelihood of available data. This model can then be used to conduct simulations of the potential impacts of alternative policy interventions across the joint distribution.

This is the approach taken in this paper to analyze the impact that pensions and income protection policies in Brazil have on contribution densities, retirement ages, and program costs. The model is based on the standard inter-temporal utility maximization framework often used to analyze policy issues related to savings and pensions reform (see Kotlikoff, 2000 for a review). In our case, the model takes into account both the pensions and unemployment benefits system (which introduces uncertainty) and, beyond savings and retirement decisions, it endogenizes choices about formal vs. informal sector work. More precisely, the level of effort invested in finding and keeping formal sector jobs.

An important difference with prior work is that instead of “calibrating” the model, we *sample* the joint distribution of model parameters to match the time distribution of a representative cohort of males (living in urban areas) across three states: contributing to the social security; outside of the social security (i.e., informal sector); or retired. Given the joint distribution we are then able to explore behavioral responses to policy changes. Our focus is on policies that introduce actuarially fair benefit formulas (expected to be incentive neutral; see Whitehouse, 2009) coupled with explicit subsidies for low-income groups. Given computational constraints, however, the analysis ignores the general equilibrium effects of these policies. In particular, we hold constant the current tax-wedge, the interest rate, wages, and labor demand.

The paper is organized as follows. Section 2 introduces the Brazilian social insurance system and discusses some key stylized facts about labor market dynamics. Section 3 introduces the model and explains the methods used to solve the inter-temporal optimization problem and perform simulations. Section 4 describes the strategy used to sample the posterior joint distribution of model parameters and assess convergence. Section 5 analyzes model dynamics under the status-quo and looks at the marginal impact of each of the model parameters on optimal contribution densities and retirement ages. Section 6 and 7 then present the results of the policy simulations and summarize the main insights from the analysis and its limitations.

## 2 The Brazilian social insurance system and labor market dynamics

Brazil spends around 12 percent of GDP on social insurance programs, which are managed by the National Social Security Institute (INSS) and the Ministry of Labor through the *Caixa Econômica Federal* (CEF). The INSS covers private sector workers and provides old-age, disability and survivorship pensions (RGPS benefits), insurance for work accidents, various transfers related to maternity and sickness leave, as well as non-contributory transfers to the poor elderly and disabled. The CEF manages the unemployment insurance system and the Length of Service Guarantee Fund (FGTS); the latter is a mandatory system of funded unemployment individual savings accounts. The RGPS is financed by pay-roll taxes (20% for most employers) and social security contributions (8% to 11% depending on the income level).<sup>2</sup> The FGTS also uses a pay-roll tax of 8% and in addition a dismissal fine of 40% of accumulated assets that is deposited into employees' FGTS account and that can only be cashed on dismissal or other few exceptions. Unemployment insurance benefits, on the other hand, are financed by the proceedings of a 0.65% tax on gross revenues (case of the services sector) and a 1.65% tax on value added (case of the industry sector).

The RGPS is quite complex; there are in fact three regimes that depend on the retirement age and the vesting period: (i) retirement based on a minimum age (53M/48W) and a minimum number of years of contributions (30M/25W) that pays a so called Proportional Length of Contribution (PLOC) Pension; (ii) retirement based on a number of years of contributions (35M/30W) and no minimum age that pays a full Length of Contribution (LOC) Pension; and (iii) retirement based on age (65M/60W) and a minimum number of years of contributions (15M/15W) that pays an Aging Pension. In all cases, the pension system guarantees a top-up so that the minimum pension (*Piso Previdenciário*) is equal to the minimum wage.<sup>3</sup> Pensions are indexed by inflation, but it is worth mentioning that in recent years the minimum wage had a real increase. The resulting replacement rates for the median and the average full-career workers can vary

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<sup>2</sup> Since January 2008, because there is no more CPMF, the smallest contribution rate is 8%.

<sup>3</sup> The Brazilian pension system also offers an essentially flat pension equal to the minimum wage to workers in rural areas (eligibility ages are 60M/55F) and to the elderly poor (BPC). These schemes, however, are not analyzed here. For an analysis of the impact of the rural pension on labor supply and retirement ages see Carvalho, 2008. The effects of BPC on labor supply are also analyzed in IPEA (2008). Finally, there is a ceiling of around 340% of average earnings to the employee contribution and benefits.

between 40 and over 100 percent depending on the retirement age and the vesting periods (see top-left panel of Figure 1).<sup>4</sup>

For the median worker and those with incomes below the median the system provides incentives for early retirement. Hence, the “implicit tax” resulting from delaying retirement by one year after eligibility to a pension is around 50% of earnings (see top-right panel of Figure 1). At the same time, for the median worker, flat Net Expected Life-time Earnings<sup>5</sup> indicate that the system provides weak incentives to contribute beyond the minimum necessary to be eligible for a pension (see bottom left panel Figure 1). This is in part because of the high level of the minimum pension and the fact that it is offered as a top-up (there is a 100% marginal tax on each monetary unit increase in the contributory pension). On the other hand, for workers earning the average or more, the system provides implicit subsidies if they delay retirement (see also Queiroz, 2005 and 2007 for a discussion about incentives for retirement). As a result, there is a large variation in the internal rates of return (IRR) on contributions that workers receive as a function of career histories and wage dynamics (see bottom-right panel in Figure 1). This implies considerable implicit and non-transparent redistribution. Also, in the majority of cases, IRRs are above sustainable levels. The pension system is thus accumulating unfunded liabilities that cannot be repaid out of future contributions and will require intergenerational transfers that can be regressive (see Robalino and Bodor 2008 for a discussion of the sustainable IRR of pay-as-you-go systems).

In terms of income protection, formal sector workers who lose their jobs after a certain number of months of contributions become eligible for an unemployment insurance benefit and a lump sum payment from their unemployment individual savings accounts. To be eligible for unemployment insurance workers need to have held a formal sector job (*trabalho with carteira*) for at least 6 months in the previous 36 month period to the start of the unemployment spell. The duration of the benefit ranges between 3 and 5 months depending on the contribution period. With 6 to 11 months workers receive 3 months of benefits, with 12 to 23 they receive 4 months, and with 24 to 36 they receive 5 months. The benefit itself depends on earnings and ranges between R\$350 in 2006 (or around 40% of average earnings) and R\$ 654.85. At the same time, workers receive a lump sum equal to the balance accumulated in their FGTS accounts while working in their last job *plus* a dismissal fine equal to 40% of the accumulated assets. As previously mentioned, the accumulations are financed with an 8% contribution rate that over a 12 months period yields a capital more or less equal to one month of salary.

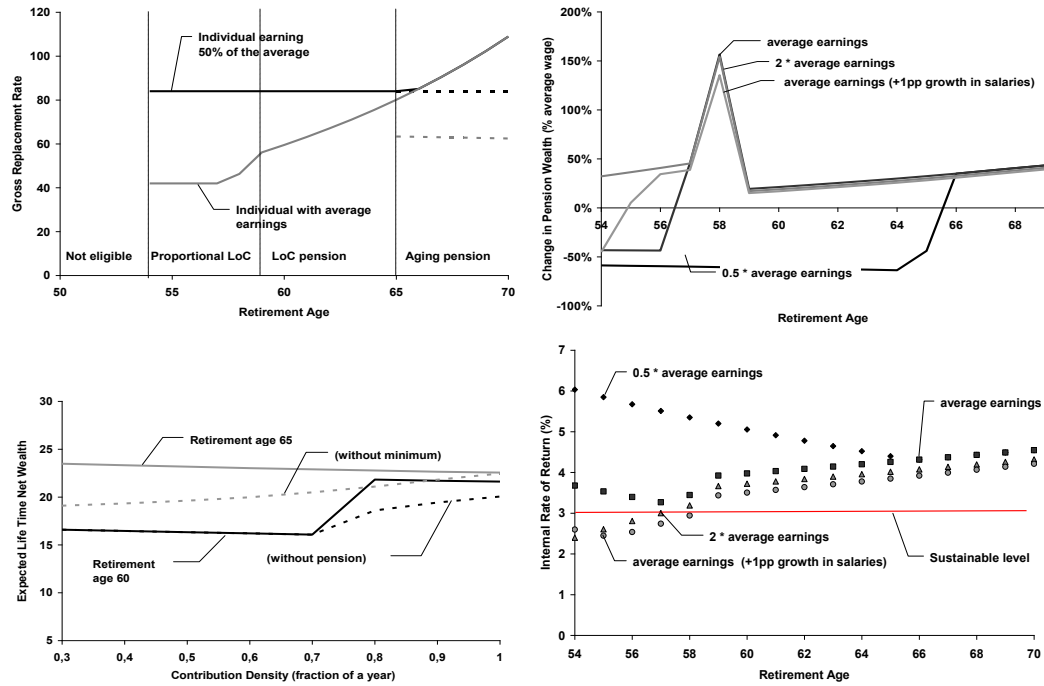
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<sup>4</sup> See Annex 1 for a description of benefit formulas for both pensions and UI.

<sup>5</sup> Net Expected Life Time Earnings are the present value of labor income and pensions over the life cycle.



Figure 1: Replacement Rates, Incentives and Redistribution in the RGPS



*Note:* The change in pension wealth (the present value of future pensions) is computed at each retirement age. A negative value indicates individuals loose pension wealth if they delay retirement by one extra year. Expected life time wealth is defined as the present value of all income flows (wages and pensions). In the calculations it is assumed that earnings in the informal sector (i.e., when the individual is not contributing to the social security represent 80% of earnings in the formal sector).  
*Source:* Authors' calculations based on current legislation.

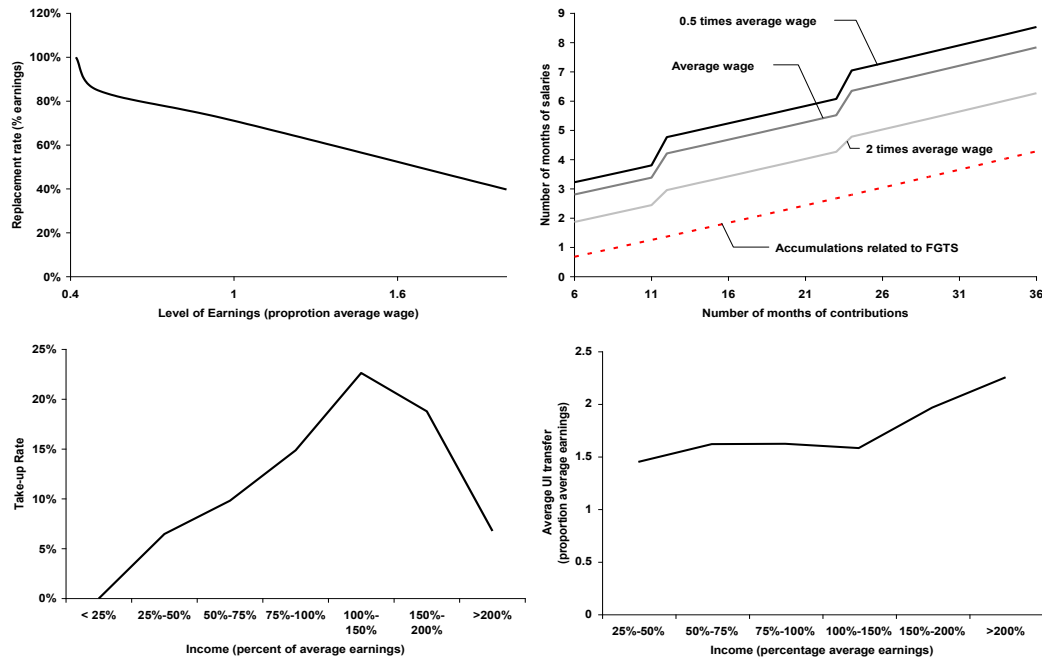
Overall, the replacement rates offered by the UI system range between 40 and 100 percent depending on the level of income. The benefit formula ensures that replacement rates are higher for low than for high income workers (see top-left panel of Figure 2). The duration of benefits is also higher for the median worker and below. Taking both UI and FGTS together, the median worker can finance between 3.5 and 8 months of salaries depending on the number of months of contributions (see top-right panel of Figure 2). Still, redistribution within the system seems to be regressive as low income workers have lower take-up rates and lower average benefits (see bottom panels of Figure 2).

In terms of incentives the evidence is somewhat mixed. The most recent analysis suggests that UI does not have a major impact on the duration of unemployment spells and, if anything, it is allowing workers to find better jobs (Margolis, 2008). Previous analysis also found that UI does not affect significantly unemployment spells, except for those transiting into self-employment. Spells in this case are shortened (see Cunningham, 2002).

Regarding FGTS, the main concern is that it is providing incentives for fake dismissals as workers attempt to cash-out their unemployment savings accounts and/or employers prefer short-

term contracts to avoid paying the dismissal fine (see Barros et al., 1999; and Gonzaga, 2003). This can happen if the rates of return on FGTS savings are consistently below market, if the mandate for precautionary savings is too high and/or there are credit constraints that impede dissavings. In our analysis, however, the focus will be on the effect of FGTS on contribution densities and retirement ages.

Figure 2: Mandate of the Income Protection System and Redistribution

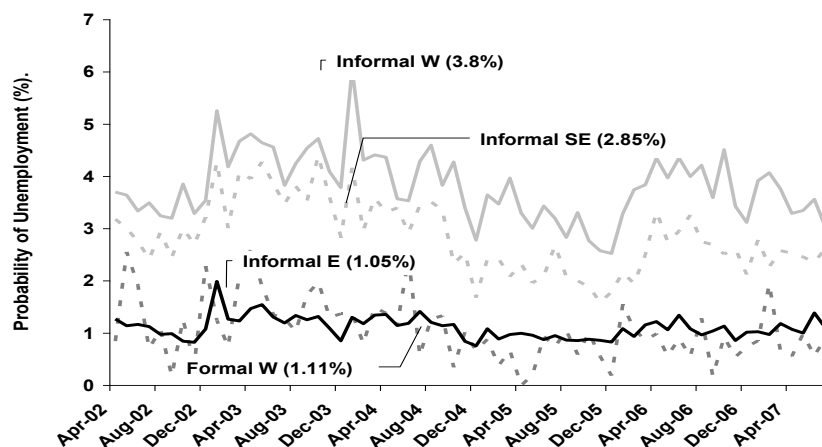


Source: Authors' calculations based on legislation and PME survey (Panel Survey of Workers in Metropolitan Areas).

In terms of general labor market dynamics, there is evidence of considerable labor mobility in Brazil and the existence of a labor market that is not fully segmented. The average duration of formal sector jobs is around 4.5 years, while the duration of self-employment and informal sectors jobs is respectively 2.3 years and a little less than one year (see Bosh and Maloney, 2007). Unemployment risks are significant, particularly among low income and informal sector workers (see **Figure 3**). There is also evidence of considerable mobility between formal and informal sectors, with flows being often symmetric after controlling for the likelihood of separation (see Bosh and Maloney, 2007).<sup>6</sup> For instance, in a given year, 42.8 percent of informal sector workers who separate from their jobs will transit to a formal sector job while 31 percent of formal sector workers will transit to an informal sector job.

<sup>6</sup> In Brazil informal sector jobs are mainly referred to jobs without social security coverage. Workers in the informal sector are thus workers *Sem Carteira*, a card that is issued by the Ministry of Labor.

Figure 3: Unemployment Risks in Brazil



Source: Authors' calculations based on PME.

### 3 The dynamic stochastic behavioral model

We are interested in formalizing the effect of the social insurance system on three economic decisions: (i) the level of savings; (ii) efforts to preserve/find jobs in the “formal sector” (defined by access to social security); and (iii) retirement decisions. We use as our starting point the standard life-cycle utility maximization framework and introduce uncertainty in employment status and life expectancy. Clearly, the assumptions of this framework are controversial; individuals are usually not fully rational and do not have perfect foresight. If they were, there would not be a need for a social security system in the first place. The model/framework is nonetheless useful as a benchmark to understand the *direction of change* in certain behaviors as a response to change in the rules of the social security system. Moreover, in our application, we explore a large range of possible behavioral responses to policy changes. Behaviors that are more like to have generated actual observations receive a higher weight. In essence, we use the model as a data generation mechanism and from this point of view it is not very different from linear single-equation econometric models.

The dynamic stochastic problem that representative individuals are assumed to solve is formally given by:

$$\begin{aligned}
& \underset{c^*, q^*, R^*}{Max} : \sum_{t=a}^{R^*} U(c_t^*, \widehat{l}_t^*; q_t^*) v_t \rho^t + \sum_{t=R^*+1}^X U(c_t^*, h) v_t \rho^t \\
& s.t. \quad k_t = k_{t-1}^* (1 + r_t) + y_t - c_t^* \\
& \quad y_t = w_t (h - \widehat{l}_t^*) (1 - \beta) e_t + [\delta_u w_t (h - l_t^*) \eta_u + S_u(\{\mathbf{Z}\}_a^{t-1}, e_{t-1}; \Psi_u)] (1 - e_t) \text{ if } t \leq R^*, \quad (1) \\
& \quad y_t = S_p(\{\mathbf{Z}\}_a^{t-1}; \Psi_p) + \delta_p w_t (h - l_t^*) \eta_p \text{ if } t > R^* \\
& \quad P(e_t = 1 | e_{t-1} = j) = 1 - \exp(-\varphi_j q_t^*), \quad j \in \{0, 1\}; \quad q \in [0, 1] \\
& \quad k_X = 0
\end{aligned}$$

where  $U(\cdot)$  is a standard utility function capturing the trade-off between consumption ( $c$ ) and leisure ( $l$ );  $v_t$  is the probability of survival to age  $t$ <sup>7</sup>;  $\rho$  is the rate of time preference;  $y$  is income;  $w$  is the wage;  $r$  the real interest rate;  $h$  the available working time during period  $t$ ;  $e$  is equal to one if the individual is employed in a “formal sector job” and zero otherwise;  $\beta$  is the social security contribution rate (paid by the employee);  $R$  is the retirement age;  $X$  is the maximum number of years a human being can live;  $a$  is the entry age to the labor market, and  $\mathbf{Z}_t = \{w_p, e_p, r_t\}$ . The function  $S_p(\cdot)$  gives the value of the pension at retirement that depends on past wages, interest rates, career histories, as well as the parameters  $\Psi_p$  of the pension system. The model allows for work after retirement from the mandatory system. Thus, with probability  $\eta_p$ , individuals who retire work in the informal sector at a fraction  $\delta_p$  of the formal sector wage. Similar to pension benefits, the function  $S_u(\cdot)$  gives the value of unemployment benefits which also depend on past values of  $\mathbf{Z}$  and policy parameters  $\Psi_u$ .

One innovation in this model is the formalization of transitions in and out of the social security (or between formal and informal jobs). These transitions are assumed to reflect, at least in part, decisions by individuals. Clearly, depending on preferences, many workers might not want to risk formal sector jobs under any circumstance. Others, on the other hand, are more likely to weight the pros and cons of formal vs. informal jobs and choose sometimes the later. This formulation can be controversial, but seems consistent with the analysis of labor market dynamics in Brazil presented in Section 2.

Thus, we assume that transitions between formal and informal sector jobs can be modeled by a Markov-type stochastic process that depends on factors which are exogenous to the worker (i.e., that the worker cannot control or change at least in the short term) and factors that are endogenous (i.e., that the worker controls). Exogenous factors refer, for instance, to the economic

<sup>7</sup> Based on the IBGE’s mortality table.

environment that makes more or less easier finding and keeping jobs (e.g., economic growth, firms turnover rates), as well as workers characteristics (e.g., level of education, sector/region where the individual works). These exogenous factors are captured by the parameters  $\varphi_0$  and  $\varphi_1$  which give respectively the probabilities of finding a job that is covered by the social security if one is outside ( $j=0$ ) or keeping a job covered by the social security ( $j=1$ ) if one is inside. The endogenous factors are captured by the variable  $q$  which represents the “level of effort” that individuals invest in finding or keeping formal jobs. As shown in system (1),  $q$  affects directly the transition probabilities in and out of the social security. We also assume that effort is “costly” and thus utility goes down when  $q$  increases ( $dU/dq < 0$ ). In order to speed-up the algorithm that solves the model, we assume that  $q$  is bounded:  $0 < q < 1$ . When  $q=1$  (maximum effort) the Markov transition matrix regulating movements in and out of the social security system is characterized by  $\varphi_0$  and  $\varphi_1$ . When  $q=0$  individuals either do not find jobs or lose jobs with probability 1. This setup is similar to that of Hopenhayn and Nicolini (1997), although their focus is on employment/unemployment transitions.

Workers who are not covered by the social security can be working in the informal sector or unemployed. But we assume that in both cases individuals can cash their unemployment benefits. Indeed, in practice, it is very difficult to enforce that individuals receiving unemployment benefits do not work in the informal sector. Moreover, as discussed in Section 2, transitions in and out of the social security are likely to go through periods of unemployment. Here we assume that with a certain probability  $\eta_u$ , individuals who exit the social security system find jobs in the informal sector. Wages in the informal sector are a fraction  $\delta_u$  of wages in the formal sector.

For the empirical work we use the standard constant risk aversion (CARA) utility function that has been adapted to take into account the level of effort put into preserving and/or finding a job. We have:

$$U(c, l, q) = \left[ \left( c^{\alpha_1} l^{1-\alpha_1} \right)^{1-\lambda} / (1-\lambda) \right] - \alpha_2 q, \quad (2)$$

where the standard parameters  $\alpha_1$  and  $\lambda$  capture respectively relative preferences for consumption and leisure and the level of risk aversion. The new parameter is  $\alpha_2$  which can be thought to capture individual attitudes towards formal sector work. A high/low  $\alpha_2$  would indicate that workers have low/strong preferences for formal sector jobs. The formulation was mainly

chosen for simplicity. It implies a constant marginal change in utility as a result of a change in effort.<sup>8</sup>

The dynamics of the model thus depend on the vector of parameters  $\theta = \{\alpha_1, \alpha_2, \lambda, \varphi_0, \varphi_1, \rho, \delta' = \delta * \eta_p\}$  that needs to be estimated; four exogenous parameters/sequences  $(\delta_u, \eta_u, \{w_t\}$  and  $\{r_t\})$ ; and the rules of the Brazilian social insurance system. We set  $w_t = \xi \cdot W_0 \cdot (1+g)^t$ , where  $W_0$  represents economy wide average earnings in the base year and  $\xi$  captures the level of income of the representative individuals in the cohort. Then across simulations we set  $g=3\%$  and  $r=4\%$ . In addition, using the labor force survey of workers in metropolitan areas (PME) we estimate that  $\delta_u=0.83$  and  $\eta_u=0.7$ .

For a given  $\theta$  and  $\xi$  we solve the model using a dynamic programming algorithm and generate a “behaviors vector”  $\mathbf{M}_d(a, e, k, v, R | \theta, \xi)$  that gives the optimal rule for decision  $\mathbf{d} = \{q^*, c^*, R^*\}$  as a function of the age  $a$  of the individual, his/her state  $e$ , the level of assets he/she holds, the vesting period  $v$  (that is the number of years the individual has contributed to the social security), and the retirement age  $R$  (if retired). The vesting period is important because of benefit formulas in the pension system.

In the dynamic programming algorithm the vector  $\mathbf{M}_d$  has the following dimensions: 80 ages, 4 activity states, 250 levels of capital, 45 vesting periods, and up to 20 retirement ages. The optimal level of the control variables  $\mathbf{d}$  is computed recursively at every point in this space taking as given the dynamics of wages, the interest rate, the benefits provided by the social insurance system, the probabilities of being alive, and the probabilities of losing/finding a formal sector job *given* the level of effort. The four states for  $e$  are: (1) out of the social security without unemployment benefits; (2) out of the social security receiving benefits; (3) contributing to the social security; and (4) retired. We track separately being out of the social security with or without unemployment benefits to control for the fact that individuals cannot receive benefits in two consecutive periods. As for the capital “grid,” 250 points give a reasonable resolution for a maximum capital equivalent to 25 times initial average earnings, so that each grid point is equivalent to 10% of average earnings. Still, the numerical approximation results in somewhat jittery optimal savings and levels of effort as a function of capital. Thus, we also use a fourth degree polynomial to smooth the optimal values in  $\mathbf{M}_d$ .

The vector  $\mathbf{M}_d$  is then used to simulate the behaviors at age  $a$  of the representative individual across  $m$  future states of the world. Thus, we generate a new vector  $\mathbf{C}_b(a, m | \mathbf{M}_d(\cdot | \theta, \xi), \mathbf{E})$

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<sup>8</sup> In a previous version of the model we used  $U(c, l, q) = \left[ (c^{\alpha_1} l^{1-\alpha_1})^{1-\lambda} / (1-\lambda) \right] (1-\alpha_2 q)$ , but this complicates the solution of the optimization program without bringing additional insights.

where  $\mathbf{b}=\{e,q^*,c^*,k\}$  and  $\mathbf{E}$  is an  $m$ -by- $a$  vector of uniformly distributed random numbers that determine the realizations of the shocks that move individuals in and out of the social security ( $\mathbf{E}$  is fix across simulations). The vector  $\mathbf{C}_b$  can then be used to compute the probability that at age  $a$ , an individual characterized by  $\mathbf{M}_d(.|\theta,\xi)$  would be in a given state  $e$ . From  $\mathbf{C}_b$  one can also derive the distribution of other output variables of interest. We keep track of six: (i) the present value of capital accumulations at age 55; (ii) contribution densities; (iii) the average value of the pension at retirement; (iv) the present value of contributory pensions paid; (iv) the present value of explicit subsidies paid through the pension system; (v) the present value of unemployment insurance benefits; and (vi) the present value of FGTS payments.

## 4 Strategy to sample the joint distribution of model parameters

There are various ways to estimate the joint distribution of model parameters, which as usual are constrained by the type of data available and computational power. The ideal, in terms of data, would be to use individual records on career histories (see, for instance, Jiménez-Martin and Sánchez-Martín, 2006). For each individual in the sample (which determines  $\xi$ ) and for a given  $\theta$ ,  $\mathbf{C}_b(a,m|\mathbf{M}_d(.|\theta,\xi),\mathbf{E})$  would then be used to calculate the likelihood of observing his/her career path (taking wages as given) and the distribution of assets at a given age(s). The vector  $\theta$  would be estimated to maximize the likelihood of the data set. The vector  $\theta$  could also be estimated for different subgroups characterized, for instance, by level of education and gender. Unfortunately, at the time of writing, individual records are still not available. But in addition, estimating in this way would be computationally very intensive. Indeed, when all the policies are “on” solving the model for a given  $\theta$  and  $\xi$  takes around 2.5 hours.<sup>9</sup> Furthermore, we are not interested in a “point estimate” of  $\theta$  but rather on a joint distribution that allows us to explore policy impacts across a large range of possible behaviors. Otherwise, one would be assuming that preferences are more or less the same across individuals (and that preferences on various dimensions are independent) and then addressing a limited range of uncertainty (on this point see Pizer, 1996).

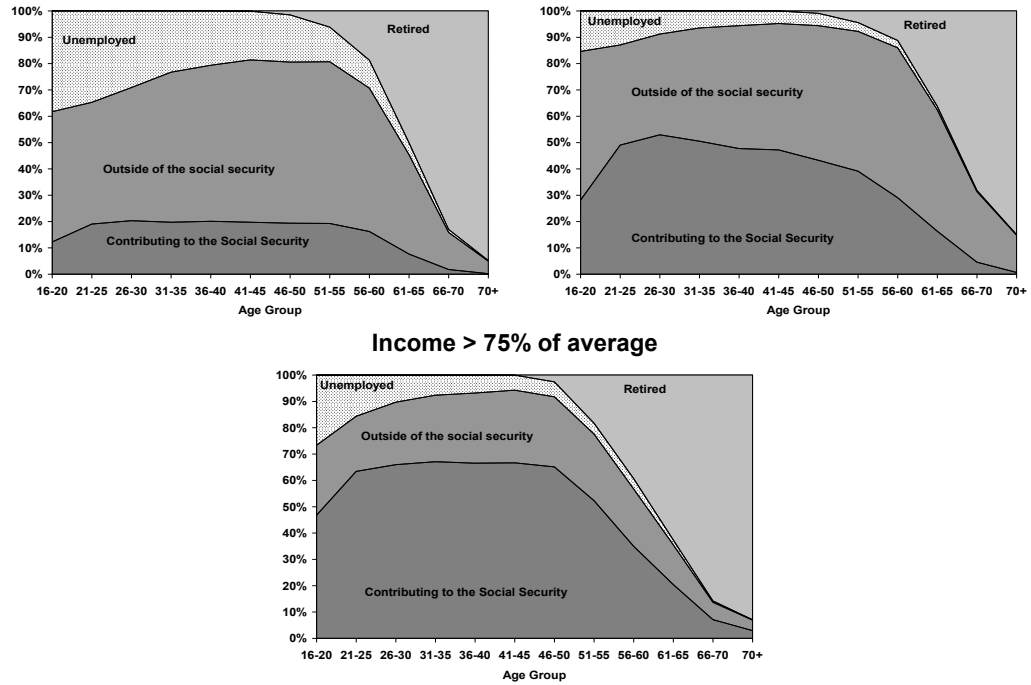
In this first application we have opted instead for a Bayesian method to sample the ex-post distribution of model parameters. In the absence of individual records we use a pseudo panel of age-cohorts derived from the *PNAD* household survey to construct a targeted distribution by state

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<sup>9</sup> We work with a server with 8 processors that operate in parallel. So, individuals could be arranged in 8 groups according to their life-time earnings (which is capture by  $\xi$ ). In this case, each iteration would take around 2.5 hours.

(contributing to the social security; outside of the social security; unemployed; and retired) for the cohort of 25 year old males who entered the labor market in year 1990 (see Annex 2). The distribution is presented in Figure 1. We focus only on urban areas and control for three levels of income: less than 50% of average earnings, between 50 and 75% of average earnings; and more than 75%.

Figure 4: Targeted Distribution for Cohort of 25 Year-Old Males in Urban Region  
Income <50% of average      Income 50% to 75% of average



Source: Household surveys 1990-2006 PNAD. The methodology to input values for ages not observed is presented in Annex 2.

The main assumption is that the aggregate distributions that we observe are the result of millions of individuals making decisions about whether to take formal or informal sector jobs, and when to retire. Some individuals, given their individual characteristics and preferences, spend most of their active lives in the formal sector. Others are most of the time outside. Yet others move in and out with more or less frequency. These various types are determined by the vector  $\theta$ . The question is, then, what is the probability of observing a given  $\theta$  given the aggregate distribution? Also, we know from Bayes rule that this probability is proportional to the probability of observing the data given  $\theta$ . So we have:

$$P(\theta | Y) \propto L(Y | \theta) f(\theta), \quad (3)$$



where  $\mathbf{Y}$  represent the aggregate distribution of the employment status by age. The goal is then to sample points from the distribution of  $\theta$  in order to maximize the likelihood of the data. Given the complexity of the model, however, we cannot sample directly from the posterior distribution. We do not have marginal distributions either that would allow us to use the Gibbs sampler (see Cassella and George, 1992). Hence, we recur instead to a more general method, the Metropolis-Hastings (MH) algorithm of which the Gibbs sampler is a particular case (see MacKay, 2003 for a presentation).

In the MH algorithm we need to assume a prior distribution for each element of  $\theta$  but the shape of this distribution does not affect the convergence properties of the algorithm, which are discussed in (Gourieroux and Monfort, 1996). Given this distribution the algorithm proceeds as follows:

1. Define  $\theta_{s=0}$ , basically our priors of the means.
2. Sample a new  $\theta'$  from a density  $f(\theta'; \theta_s)$
3. Calculate  $d = L(\mathbf{Y} | \theta') f(\theta_s, \theta') / L(\mathbf{Y} | \theta_s) f(\theta', \theta_s)$
4. If  $d > 1$  then  $\theta_{s+1} = \theta'$
5. Otherwise,  $\theta_{s+1} = \theta'$  with probability  $d$ .
6. Goto 2.

The intuition is that the means of the densities from which we sample  $\theta$  will be updated each time the likelihood of observing the data given the parameters improves. When there is no improvement ( $d < 1$ ) the mean can still be updated but with a probability that is proportional to  $d$ . If  $d$  is very low the probability that the mean is updated is also very low. We also notice that the improvement in the likelihood of observing the data is corrected by the odds of having sampled the parameters in the first place given the means of the distributions. In a symmetrical distribution such as the normal  $f(\theta_s, \theta') / f(\theta', \theta_s)$  is always equal to one. But when censoring is introduced in the distribution of certain parameters, which is our case, the correction is needed.

In our application we use independent prior normal distributions for each of the parameters with eight different initial means – hence the final distribution is based on eight independent sequences of sampled parameters. For the parameters risk aversion ( $\lambda$ ) and the time preference ( $\rho$ ) we fix the means based on references from the literature. For preferences for consumption over leisure ( $\alpha_1$ ) and formal vs. informal sector work ( $\alpha_2$ ) we allow for a more or less arbitrary

initial range of variation. For the parameters that determine transitions in and out of the social security we did some simulations to understand their influence on the steady state distribution of the cohort. On this basis we defined initial values and also imposed the constraint  $\varphi_0 < \varphi_1$  so that the probability of keeping a job is always higher than the probability of finding one (which is consistent with the data reviewed in Section 2). Finally, for the probability of working when retired we used as a starting reference the average derived from the household survey.

For some of the parameters the economic model puts restrictions on their range of variation, hence we apply left or right censoring. In all cases, judgment is involved in setting the variance of the distributions so that there is enough variation to explore larger regions of the parameter space, but not too much that it would delay convergence (see MacKay, 2003). The initial distributions of the model parameters for the 8 sequences are presented in Table 1.

To compute the ratio  $d$  for each  $\theta_i$  we proceed as follows. First we use  $\mathbf{C}_b(a, m | \mathbf{M}_d(\cdot | \theta, \xi), \mathbf{E})$  with  $m=1,000$  to compute the probabilities that at various ages  $a$  an individual of the cohort would be in various states  $e$ . We define these probabilities by  $p(a, e)$  and calculate them by simply counting the number of individuals in state  $e$  at age  $a$  and then dividing by  $m$ . Then, the probability that the data would have been generated by  $\theta$  is given by the multinomial distribution:

$$P(Y | \theta) = \prod_a \left( F_a \prod_e p(a, e)^{s(a, e)N_a} \right), \quad (4)$$

where  $N_a$  is the number of individuals of age  $a$  who were sampled from the population,  $s(a, e)$  the share of these individuals that is in state  $e$  (which come from the pseudo sample), and  $F_a$  the number of possible combinations of individuals across states. Because we are only interested in likelihood ratios, the sample size is normalized to 1 so that (4) becomes the Dirichlet distribution with parameters  $s(a, e)$ .<sup>10</sup> The ratio  $d$  is then given by:

$$d = \frac{\prod_a \left( \prod_e p(a, e | \theta')^{s(a, e)} \right) f(\theta_s | \theta')}{\prod_a \left( \prod_e p(a, e | \theta_s)^{s(a, e)} \right) f(\theta' | \theta_s)}, \quad (5)$$

where the normalizing constants for the distributions are dropped from both the numerator and denominator. Then taking logs we obtain:

$$\log(d) = \sum_a \sum_e s(a, e) \left( \log(p(a, e | \theta')) - \log(p(a, e | \theta_s)) \right) + \log(f(\theta_s | \theta')) - \log(f(\theta' | \theta_s)), \quad (6)$$

<sup>10</sup> The Dirichlet distribution is a Bayesian prior of the parameters of the Multinomial distribution. It gives the likelihood of the probabilities  $p(a, e)$  given the shares of each cohort in each state.

The only missing pieces to compute  $d$  are then probabilities of sampling the parameters given the means. Taking into account the left hand and right hand truncations and the variances of the normal distributions these probabilities are given by:

$$\log(f(\boldsymbol{\theta}' | \boldsymbol{\theta}_s)) = \sum_{\theta} \log \left\{ \begin{array}{ll} \frac{N(\theta_i | \boldsymbol{\theta}_s, \Phi_s) - N(\theta_{\min} | \boldsymbol{\theta}_s, \Phi_s)}{N(\theta_{\max} | \boldsymbol{\theta}_s, \Phi_s) - N(\theta_{\min} | \boldsymbol{\theta}_s, \Phi_s)} & \text{if } \theta_i < \theta_{s,i} \\ \frac{N(\theta_{\max} | \boldsymbol{\theta}_s, \Phi_s) - N(\theta_i | \boldsymbol{\theta}_s, \Phi_s)}{N(\theta_{\max} | \boldsymbol{\theta}_s, \Phi_s) - N(\theta_{\min} | \boldsymbol{\theta}_s, \Phi_s)} & \text{if } \theta_i > \theta_{s,i} \end{array} \right\}, \quad (7)$$

where  $N$  is the cumulative normal distribution,  $i$  indexes the elements of  $\boldsymbol{\theta}$  and  $\Phi$  is the variance covariance matrix of the prior distribution of the parameters that here is assumed to be a diagonal matrix (i.e., there are no prior correlations between the model parameters).

Table 1: Initial Distributions for the Eight Independent Samples of Model Parameters

<b>Average Earnings</b>						
	<b>Risk Aversion (<math>\lambda</math>)</b>	<b>Time preference (<math>\rho</math>)</b>	<b>Alfa 1 (<math>\alpha_1</math>)</b>	<b>Prob. Keep Formal Job (<math>\phi_1</math>)</b>	<b>Alfa 2 (<math>\alpha_2</math>)</b>	<b>Prob. Work when Retired (<math>\eta_p</math>)</b>
<b>Mean 1</b>	1,50	0,04	0,90	0,95	0,30	0,50
<b>Mean 2</b>	1,30	0,03	0,80	0,90	0,20	0,40
<b>Mean 3</b>	1,20	0,02	0,70	0,85	0,10	0,30
<b>Mean 4</b>	1,10	0,01	0,60	0,80	0,05	0,20
<b>Mean 5</b>	0,80	-0,01	0,70	0,85	0,05	0,20
<b>Mean 6</b>	0,70	-0,03	0,70	0,85	0,01	0,20
<b>Mean 7</b>	1,20	-0,02	0,80	0,85	0,10	0,30
<b>Mean 8</b>	1,20	0,03	0,80	0,85	0,05	0,30
<b>Variance</b>	0,05	0,01	0,05	0,05	0,05	0,05
<b>Trunc. left</b>	0	-99	0,5	0	0	0
<b>Trunc. right</b>	99	99	1	1	1	1
<b>50% Average Earnings</b>						
	<b>Risk Aversion (<math>\lambda</math>)</b>	<b>Time preference (<math>\rho</math>)</b>	<b>Alfa 1 (<math>\alpha_1</math>)</b>	<b>Prob. Keep Formal Job (<math>\phi_1</math>)</b>	<b>Alfa 2 (<math>\alpha_2</math>)</b>	<b>Prob. Work when Retired (<math>\eta_p</math>)</b>
<b>Mean 1</b>	1,50	0,04	0,90	0,50	0,30	0,50
<b>Mean 2</b>	1,30	0,03	0,80	0,90	0,20	0,40
<b>Mean 3</b>	1,20	0,02	0,70	0,85	0,10	0,30
<b>Mean 4</b>	1,10	0,01	0,60	0,80	0,05	0,20
<b>Mean 5</b>	0,80	-0,01	0,70	0,45	0,05	0,20
<b>Mean 6</b>	0,70	-0,03	0,70	0,85	0,01	0,20
<b>Mean 7</b>	1,20	-0,02	0,80	0,45	0,10	0,30
<b>Mean 8</b>	1,20	0,03	0,80	0,85	0,05	0,30
<b>Variance</b>	0,05	0,01	0,05	0,05	0,05	0,05
<b>Trunc. left</b>	0	-99	0,5	0	0	0
<b>Trunc. right</b>	99	99	1	1	1	1

Source: Range of variation for Risk Aversion and Time preference parameters based Jiménez-Martín and Sánchez-Martín (2006). For the other parameters see main text.

To assess the convergence of the various series we follow the method proposed in Gelman et al. (2000). The idea is to compare an over estimate and an under estimate of the posterior marginal variance of the parameters in  $\theta$  and see whether they converge. The overestimate of the variance is given by the weighted sum of the between sequences ( $B_i$ ) and within sequences ( $W_i$ ) variances for each parameter  $\theta_i$ . We have:

$$\text{var}^+(\theta_i | \mathbf{Y}) = \frac{n-1}{n} W_i + \frac{1}{n} B_i, \quad (8)$$

with

$$B_i = \frac{N}{Z-1} \sum_{z=1}^Z (\bar{\theta}_{i,z} - \bar{\theta}_{i..})^2, \quad \text{with} \quad \bar{\theta}_{i,z} = \frac{1}{N} \sum_{n=1}^N \theta_{inz} \quad \text{and} \quad \bar{\theta}_{i..} = \frac{1}{Z} \sum_{z=1}^Z \bar{\theta}_{i,z}, \quad (9)$$

$$W_i = \frac{1}{Z} \sum_{z=1}^Z \sigma_{iz}^2, \quad \text{with} \quad \sigma_{iz}^2 = \sum_{n=1}^N (\theta_{inz} - \bar{\theta}_{i,z})^2$$

where  $Z$  is the number of independent sequences and  $N$  the number of samples in each sequence. Both,  $B_i$  and  $W_i$  overestimate the marginal posterior variance if the initial distribution is appropriately over dispersed, but the estimator is unbiased when  $n$  is large ( $n \rightarrow \infty$ ).

For a finite  $n$ , however, the within variance ( $W_i$ ) should be an underestimate because the individual sequences have not had yet time to range over all the targeted distribution and therefore have less variability. Then an indicator of the potential gains of continuing with the iterations is:

$$\sqrt{\hat{R}_i} = \sqrt{\frac{\text{var}^+(\theta_i | \mathbf{Y})}{W_i}}, \quad (10)$$

If  $R_i$  is equal or close to one the series have converged. For applications like ours where we are less interested in the precision of the posterior joint distribution but care more about taking into account sufficient heterogeneity in behaviors, we consider values up to 1.2.

We applied this methodology to derive the posterior distribution for individuals with average earnings (we will refer to them as “high” income) and those with earnings equal to 50% of the average (“low” income). The main descriptive statistics for each of the parameters are presented in

**Table 2.** It is important to emphasize, however, that the numbers taken independently do not mean much. What matters are the *various combinations of model parameters* that make the joint distribution. Still, it is instructive to see that the statistics reported are consistent with our priors and other results in the literature. The coefficient of risk aversion, for instance, has an

average of 1.2 for both low and high income individuals, indicating that most people are risk averse. We also confirm negative or low levels for the average rate of time preference. For high income workers the median rate is 0.1% and for low income workers 0.4%. This is consistent with the results in Jiménez-Martín and Sánchez-Martín (2006) showing that in the absence of social security individuals will tend to retire late. The distributions also suggest stronger preferences for consumption over leisure for both high and low income workers. In addition, not surprisingly, higher income workers have a higher exogenous probability of formal work than low income workers and face a lower disutility of keeping and finding formal sector jobs. Finally, the distributions indicate that work after retirement is common, particularly for high income workers. Or, in other words, the model is more likely to generate predictions consistent with the aggregate distribution when individuals are assumed to work after retirement.

Table 2: Joint Distribution of Model Parameters

	<b>Risk Aversion (<math>\lambda</math>)</b>	<b>Time preference (<math>\rho</math>)</b>	<b>Alfa 1 (<math>\alpha_1</math>)</b>	<b>Prob. Keep Formal Job (<math>\phi_1</math>)</b>	<b>Alfa 2 (<math>\alpha_2</math>)</b>	<b>Prob. Work when Retired (<math>\eta_p</math>)</b>
<b>Average earnings</b>						
Mean	1.2522	-0.0092	0.8039	0.8742	0.1003	0.4156
Standard Error	0.0125	0.0037	0.0142	0.0077	0.0090	0.0188
Median	1.2430	0.0013	0.8273	0.8786	0.0760	0.4400
Minimum	1.0732	-0.0828	0.5045	0.6833	0.0040	0.0976
Maximum	1.5405	0.0512	0.9895	0.9817	0.3224	0.7213
<b>50% Average Earnings</b>						
Mean	1.2169	0.0099	0.7374	0.7529	0.1854	0.3344
Standard Error	0.0260	0.0035	0.0158	0.0259	0.0124	0.0221
Median	1.2106	0.0048	0.7333	0.8614	0.1791	0.3351
Minimum	0.6345	-0.0525	0.5065	0.4284	0.0177	0.0689
Maximum	1.5474	0.0727	0.9916	0.9821	0.3962	0.6293

Source: Authors' calculations.

In terms of the convergence statistics, we obtain for most parameters R values close to 1 (see **Table 3**). The only exception is the coefficient of risk aversion. The R value of 1.4 suggests that further iterations with the MH algorithm would have narrowed the variance of the distribution. Still, as discussed above, both the current average and median of the risk aversion coefficient for high and low income workers are consistent with other results in the literature.

Table 3: Convergence Statistics for Various Parameters

<b>Average Earning</b>				
<b>Parameter</b>	<b>B<sub>i</sub></b>	<b>W<sub>i</sub></b>	<b>Var(<math>\theta_i Y</math>)</b>	<b>R<sub>i</sub></b>
Risk Aversion ( $\lambda$ )	4.046	0.221	0.322	1.454
Time Pref. ( $\rho$ )	0.033	0.009	0.010	1.070
Alfa 1 ( $\alpha_1$ )	0.493	0.142	0.151	1.065
Prob. Formal ( $\phi_1$ )	0.232	0.337	0.334	0.992
Alfa 2 ( $\alpha_2$ )	0.319	0.141	0.146	1.033
Prob. Working ( $\eta_p$ )	1.515	0.156	0.191	1.230
<b>50% Average Earning</b>				
<b>Parameter</b>	<b>B<sub>i</sub></b>	<b>W<sub>i</sub></b>	<b>Var(<math>\theta_i Y</math>)</b>	<b>R<sub>i</sub></b>
Risk Aversion ( $\lambda$ )	2.736	0.159	0.227	1.427
Time Pref. ( $\rho$ )	0.052	0.008	0.009	1.154
Alfa 1 ( $\alpha_1$ )	0.310	0.126	0.130	1.039
Prob. Formal ( $\phi_1$ )	1.660	0.169	0.208	1.232
Alfa 2 ( $\alpha_2$ )	0.140	0.155	0.154	0.998
Prob. Working ( $\eta_p$ )	0.816	0.124	0.143	1.146

Source: Authors' calculations.

## 5 Dynamics under the Status-quo

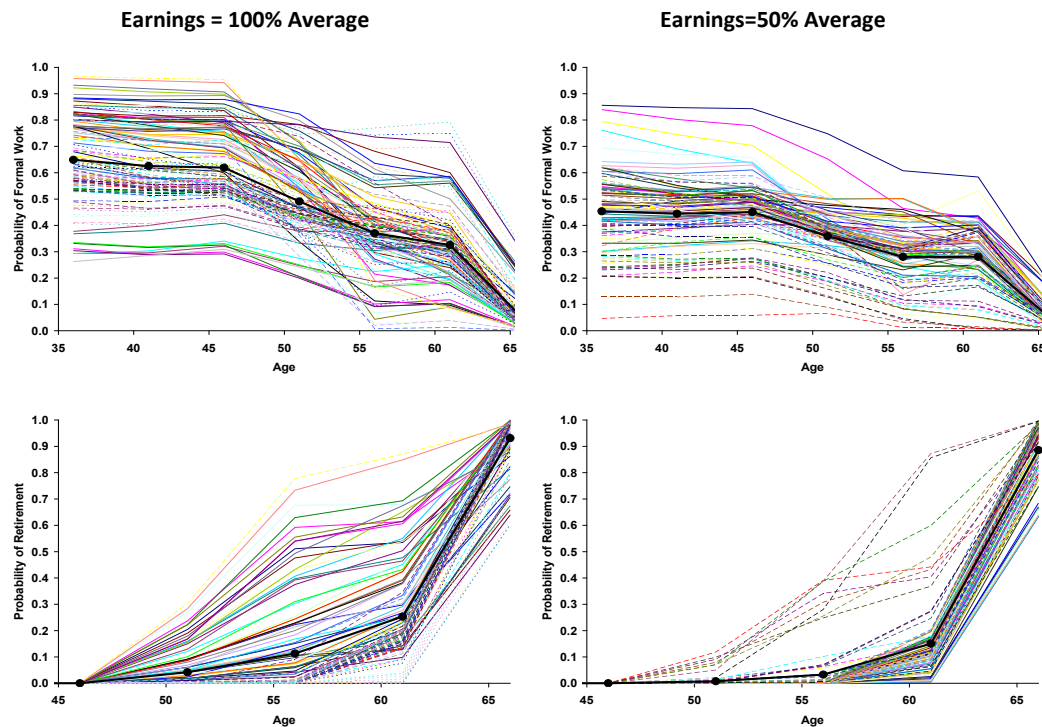
For each income level (“high” and “low”) we run the model “across” the joint distribution of parameters focusing on three outcomes: the probability that an individual is contributing to the social security at a given age, the probability that the individual is retired at a given age, and assets accumulations by age 55.

The results regarding the probability of contributing to the social security or being retired at a given age are summarized in **Figure 5**. Each line refers to one point of the joint distribution of model parameters. As in the estimation, the probabilities are computed based on 1,000 runs of the model. The figure shows that, under the status-quo, on average, around 30-35 percent of the high-income workers and 45-50 percent of the low income would be outside of the formal sector between ages 35 and 45. Afterwards, the probability of formal sector work declines for both high and low earners. This is consistent with the current distribution of age cohorts as discussed in Section 4. Thus, for several sets of preferences, we find the pattern of a declining probability of formal work with age which is found in the empirical analysis of labor market transitions over the life-cycle (see Perry et al. 2007; and Robalino et al., 2009). The standard interpretation is that, with age, workers gain experience and easier access to credit, and that many then prefer to switch to self-employment. In our model we do not formalize experience (other than through the real growth rate of wages) and individuals are not allowed to borrow. The interpretation instead is that individuals will have the *motivation* to move out of formal sector jobs as they get older. In

essence, the “marginal utility” of formal jobs relative to informal jobs goes down with time – while the marginal disutility linked to the effort invested in finding and keeping jobs remains constant (given the shape of the utility function). The main reasons for this are higher consumption levels and higher asset accumulations. In our setting, therefore, it becomes optimal to reduce efforts in finding/keeping formal sector jobs.

In terms of retirement, the model predicts that around half of the high earners would retire between ages 55 and 60. Low income individuals, on the other hand, tend to retire later – between ages 60 and 65. This is also consistent with the analysis of cohorts presented in Section 4 and the micro-data analyzed in World Bank (2008). But again, the variation in retirement patterns can be considerable. Some individuals can retire as early as 53, others can delay retirement until 70 (see bottom two panels of **Figure 5**).

Figure 5: Probabilities of Contributing to INSS and Retiring



Note: The dark lines with dots give the “average” path for the cohort.  
Source: Simulation model.

Overall, the results emphasize the significance that individual preferences have in determining behaviors – and therefore the impact of alternative policies. It is thus important to understand what is the marginal effect that various parameters have on outcome variables of interest, in this case contribution densities and retirement ages. To do this we estimated stepwise

regressions of these two variables on the six model parameters, their squares, and their interactions (15 regressors in total). For the estimation we used the entire joint distribution of model parameters which has around 550 points.

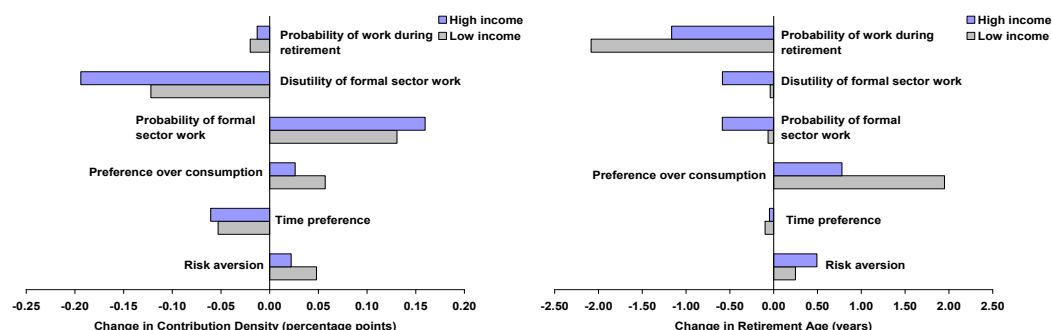
The results are summarized in Annex 3 and show, not surprisingly, that the model parameters affect the endogenous variables through complex interactions. The resulting *linear approximations* (or second order expansions) of the structural model differ for low and high income workers – which is consistent with the fact that the social security system affects low and high income workers differently. But the sign of the partial derivatives of the endogenous variables with respect to each of the parameters is the same for both high and low income workers. In all cases, the signs are consistent theoretically and intuitively (see **Figure 6**).

In terms of contribution densities, the parameters that have a positive effect are: the coefficient of risk aversion (more risk adverse individuals demand more insurance), the preference for consumption over leisure (preference towards consumption provides incentives to increase earnings through formal sector work), and the exogenous probability of formal sector work (other things being equal the higher this probability the higher the contribution density). On the other hand, the parameters that reduce contribution densities are: the rate of time preference (the more individuals discount the future the less willing they are to invest in long term savings), the disutility of efforts to find/keep formal sector jobs (other things being equal, the higher the disutility the least effort individuals invest in joining/staying in the social security), and the probability of working when retired (the higher the expected value of this source of income the lower the incentives to contribute for pensions).

Regarding retirement ages, the parameters that have a positive impact are the coefficient of risk aversion (more risk adverse individuals prefer to increase earnings and savings and delay retirement), and preferences over consumption (which also provide incentives to increase earnings and delay retirement). All the other parameters have a negative effect. The more individuals discount the future the less willing they are to defer cashing-up their pensions; the higher the probability of formal work the sooner the individual can meet eligibility conditions for a pension (and would also have accumulated higher savings); the higher the disutility of formal sector work the lower the incentives individuals have to keep working; and, finally, the higher the probability of working while retired the lower the forgone revenues from retirement.



Figure 6: Partial Derivatives in Second Order Expansion of the Structural Model

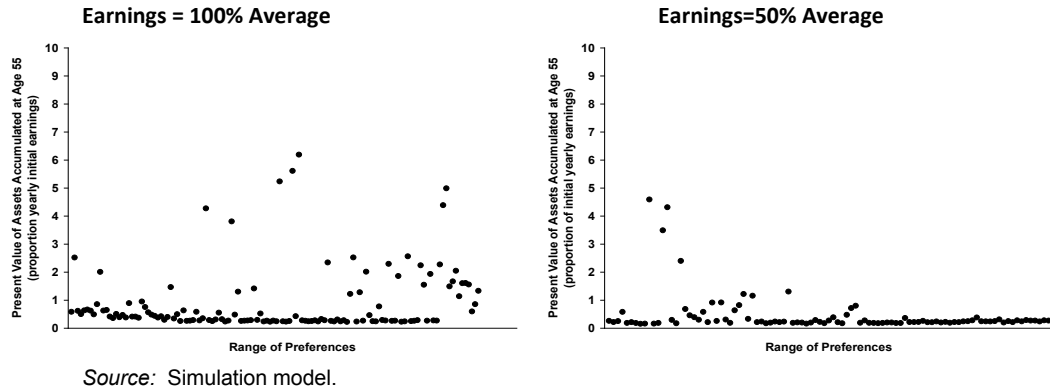


Source: Simulation model.

The simulations also predict average asset accumulations by age 55. For those with earnings equal to the average the present value of assets accumulated by age 55 is equivalent, on average, to 90 percent of initial yearly earnings. For those with earnings equal to 50 percent of the average, accumulations represent, on average, 45 percent of initial yearly earnings.<sup>11</sup> These predictions have not been compared with real data but the order of magnitude is not disparate. In general, the results suggest low levels of savings. But savings also vary considerably depending on preferences (see Figure 7). Among average earners, the individual who saves the least would have assets worth, in present value, less than three months of initial earnings, while the individual who saves the most would have savings representing 6 times initial yearly earnings. The lowest and highest level of savings among low income workers are respectively one month and 4.5 years of earnings. We did not estimate a linear model to look at the marginal effect of each parameter on savings rates but simple correlations show that the main parameters influencing savings are the coefficient of risk aversion and preferences for consumption over leisure. In the rest of the analysis, however, the focus will be on contribution densities and retirement ages.

<sup>11</sup> These assets exclude the pension wealth from the mandatory system but include accumulations in the FGTS program.

Figure 7: Individual Preferences and Assets Accumulations



## 6 Potential Impact of Policy Changes

We start by looking at the marginal impact of each of the programs on retirement ages and contribution densities “across” the joint distribution of parameters. We basically ask the question what would be the impact on the output variables of interest of removing, one at the time, the pension system, the unemployment insurance system, and the unemployment FGTS savings accounts. We then look at the effect of policy interventions that aim to separate the insurance and redistributive functions in the pensions system. This is done by having one single formula for pensions that is “incentives neutral” and then using various forms of explicit subsidies to finance transfers for targeted individuals.

A general, and important, message from the analysis is that, like in the case of the baseline, the effects of any policy intervention on behaviors are very sensitive to individual preferences. One could compute an average effect for each intervention, for instance, an average increase or reduction in contribution densities and retirement ages. But this average effect would hide considerable variation resulting from unobserved heterogeneity in preferences. Thus, in what follows we look at the impact of policy changes on a sub-region of the parameters space, basically the 35 points with the highest likelihood or the “center” of the joint distribution. We limited ourselves to 35 points mainly given constraints in terms of computing time.

We first remove the pension system. The effects on contribution densities<sup>12</sup> and retirement ages are quite different for high and low income workers. For high income workers we observe, in

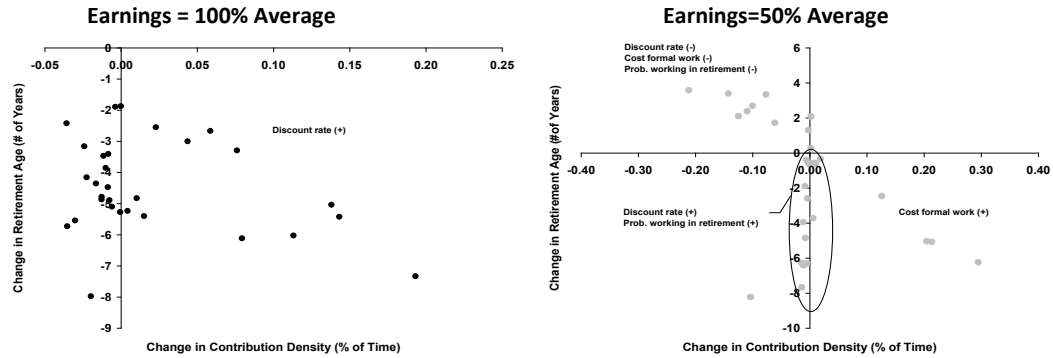
<sup>12</sup> Although there is no more pension system and, therefore, no more individual contributions, we still consider formal work to be contributory, because of the taxes paid to finance UI and the FGTS contribution paid by the employer.

most cases, a reduction in retirement ages and an increase in contributions densities (see top left panel in Figure 8). Basically, the pension system as it is provides incentives to high income workers to delay retirement but reduces incentives for formal work. In Section 2 we had already pointed out that the Brazilian pension system pays implicit rates of return (IRR) on contributions above market and that the IRR goes up with the retirement age. Hence, it is not surprising to see retirement ages going down when the pension system is eliminated. At the same time, the higher rates of return on contributions have an income effect that allows workers to contribute less (and in fact save less) for retirement. Thus, when the pension system is eliminated we do observe people spending more time in the formal sector and saving more (see bottom panel in Figure 8).

For low income workers there is more variation in the behavioral response. In around 1/3 of the cases retirement ages increase and contribution densities fall. In a few cases the opposite occurs and, for the majority, contribution densities remain more or less unchanged while retirement ages fall (see top right panel of Figure 8). To interpret these results it is useful to think about individuals having *natural* retirement ages and contribution densities – those that would be observed without the pension system. As we showed in the previous section, there would be a large variation in these retirement ages as a function of individual preferences. A first group of individuals would naturally opt to retire late and participate less in formal sector work (as shown in Section 2 less formal work is correlated with delayed retirement). But because of the pension system and its implicit (the high IRR) and explicit subsidies (the minimum pension), these individuals can afford to advance retirement. At the same time, to be eligible for the minimum pension guarantee, they are willing to put more effort into finding/keeping formal sector jobs. Thus, in the simulations, when the pension system is eliminated, these individual (preferences) appear in the north east quadrant of the figure: they delay retirement and participate less in formal sector work.

A second group of individuals tends to retire early, in part for instance, as a result of a higher exogenous probability of finding/keeping formal sector jobs where earnings are higher. Hence, they also have higher contribution densities. Because of the pension system, however, they have an incentive to delay retirement to benefit from the minimum pension guarantee. They can also afford to reduce contribution densities (which are “*naturally high*”) since higher vesting periods will not imply higher pensions (due to the 100 effective marginal tax rate discussed in Section 2). When in the simulations the pension system is eliminated, these individuals appear in the south-east quadrant: they reduce retirement ages (since there is no longer a minimum pension as an incentive) and they increase contribution densities.

Figure 8: Effects of Removing the Pension System

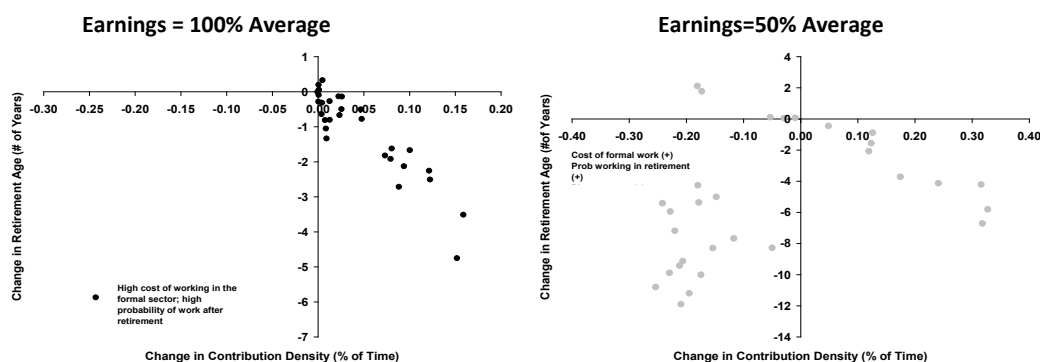


Source: Simulation model.

Finally, in the largest group, there are individuals for whom the pension system provides incentives to delay retirement (because of eligibility for the minimum pension) but who do not have high contribution densities (for instance given a high probability of work during retirement or a high disutility of formal sector work) and thus do not reduce them (otherwise they would not meet the eligibility conditions). When the pension system is removed these individuals appear within the ellipse of the chart: they reduce retirement ages but leave contribution densities more or less unchanged.

Regarding the unemployment insurance system there are two effects that one could expect. For some workers removing UI would provide more incentives to self-insure – which implies spending more time in the formal sector and saving more. For others, eliminating UI implies losing implicit subsidies that are financed entirely by the employer and this reduces incentives to enroll in the social security. For high income workers the first effect dominates – there is only one case where the contribution density falls which is associated with a high disutility of formal employment. When losing UI these workers contribute more and then can also afford to retire earlier. For low income workers in half of the cases the same phenomenon is observed and contribution densities increase. In the other half of the cases, however, contribution densities decrease. These cases are characterized by a high disutility of formal work, a high discount rate, and a high probability of work during retirement. And indeed, as discussed in the previous section, the first two parameters tend to have higher values among low income workers. For these individuals eliminating UI reduces the incentives they have to engage in formal sector work which is *naturally* costly.

Figure 9: Effects of Removing the Unemployment Insurance System



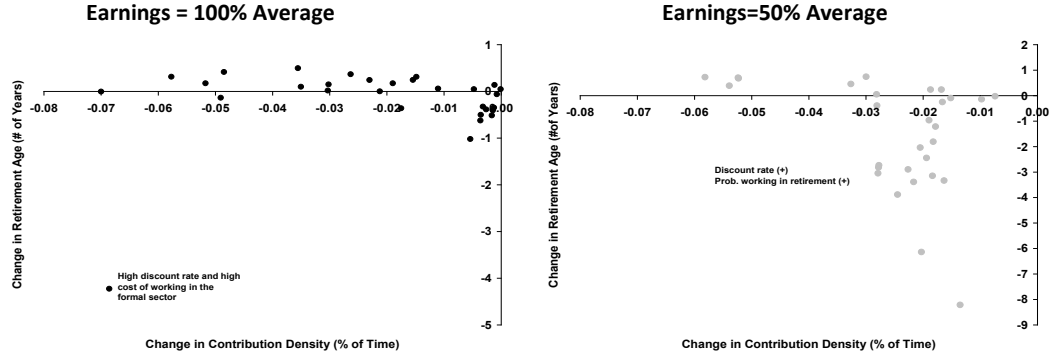
Source: Simulation model.

The effects of FGTS also differ for high and low income workers. For high income workers the main effect is a reduction in contribution densities without a meaningful change in retirement ages (see left panel of **Figure 10**). Lower contribution densities are not surprising. Indeed, eliminating the program reduces a substantial share of subsidized savings – since contributions are paid by the employer – and therefore incentives to contribute to the social security.<sup>13</sup> We also notice, however, that there is a group of high income individuals for whom contribution densities do not change. The main interpretation is that contribution densities for them are “binding;” reducing them further would make them ineligible for the highest pension at a given retirement age.

For low income workers we also observe a set of preferences for which a drop in contribution densities occurs without meaningful changes in the retirement age – with or without FGTS they most likely retire late. A majority of individuals, however, choose to retire earlier when FGTS is eliminated (see right panel of **Figure 10**) without changing much contribution densities. The interpretation is that when losing FGTS these individuals have fewer incentives to delay retirement and continue benefiting from subsidized savings. Clearly, the additional savings from FGTS can also allow some workers to retire earlier, but the first effect in this case dominates.

<sup>13</sup> Clearly, FGTS can also induce fake dismissals and promote informal sector work as workers tend to cash savings. But this does not imply reducing the contribution density. Hence, higher contribution densities can co-exist with a higher turn over rate (which given the long-term nature of our model we have not computed).

Figure 10: Effects of Removing FGTS



Source: Simulation model.

Next we look at the effects of possible reforms that introduce “incentive neutral” benefit formulas in the pension system and make redistribution explicit and targeted to individuals with limited savings capacity. The policy changes that we simulate are summarized in Table 4. The common feature, in all cases, is that the benefit formulas for pensions are unified into one given by:

$$p_R = \frac{\beta_w + \beta_e + \beta_g}{G_R(irr)} \left( \sum_{i=a}^R w_i \cdot (1 + irr)^{R-i} \right), \quad (11)$$

where  $p_R$  is the pension paid by the system at retirement age  $R$ ;  $\beta_w$ ,  $\beta_e$ , and  $\beta_g$  are the contribution rates paid to the system respectively by the employee, the employer and the government (when there are explicit subsidies);  $a$  is the age when the individual joins the system,  $irr$  is the rate of return that the system pays on contributions; and  $G_R(irr)$  is an annuity factor that also depends on  $irr$ . In our application,  $irr$  is assumed to be equal to the growth rate of the average wage, which as shown in Robalino and Bodor (2008) is a good proxy to the sustainable internal rate of return of a pay-as-you-go system (although in most cases this proxy would be below the sustainable rate).

In terms of contribution rates, we assume that employees pay 8% (equal to the minimum contribution rate today) and that out of the 20 percentage points paid by the employer, 8 percentage points are allocated to finance old-age pensions. Thus the total contribution rate to finance pensions is 16%.

Table 4: Summary of Policy Interventions

REFORMS	DESCRIPTION
Reform 1	Pensions benefit formulas are unified. Eligibility age for pension is fixed at age 55. There is no vesting period and no minimum pension guarantee. No changes in UI and FGTS.
Reform 2	Like Reform 1 but a minimum pension guarantee equal to 42% of economy wide average earnings is offered at age 55 as a top-up (100% marginal tax).
Reform 3	Like Reform 2 but the minimum pension guarantee is only offered at age 65.
Reform 4	Like Reform 3 but the minimum pension is offered as a flat rate (0% marginal tax).
Reform 5	Like Reform 1 but a matching contribution equivalent to 75% of the total contribution is offered and financed by the government. Two retirement ages are explored: 55 and 65.
Reform 6	Like Reform 5 but the matching contribution is equivalent to 2.25 times the total contribution. Two retirement ages are also explored: 55 and 65.

Source: Authors.

We first analyze the case where the pension formula is unified, the eligibility age becomes 55, the vesting period is eliminated, and there is no minimum pension (see **Figure 11**). This scenario tells us what would happen in a pension system that is actuarially fair (and financially sustainable) and where there are no restrictions on retirement.

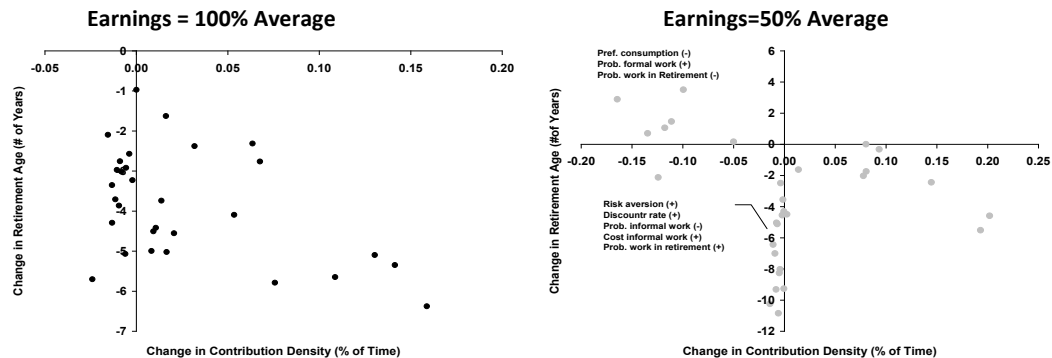
We observe that for the average earner the results are similar to the case where the pension system is eliminated. In essence, relative to the status-quo, these individuals lose subsidies to retire late. Their optimal reaction is therefore to reduce retirement ages and increase contribution densities (and savings). Relative to the case with no pension system (**Figure 8**) there is a small increase in contribution densities and retirement ages. Basically, for average earners, adding an actuarially fair pension system where the employer matches the contribution rate can increase marginally the time individuals spend in formal sector work. And, in most cases, there are no major impacts on the retirement age, although one can observe a few cases where the retirement age increases (waiting to retire increases the value of the pension) and others where it decreases (pensions allow individuals to retire early).<sup>14</sup>

For low income workers the results relative to the status-quo are also similar to the case with no pension system: retirement ages and contribution densities can increase or decrease depending on individual preferences. When we compare the results to the case without pension system we also see that retirement ages and contribution densities can go up or down. The

<sup>14</sup> Although not shown, in most cases, the actuarially fair pension system with a matching contribution from employers would decrease savings.

matching from the employer provides more incentives to contribute and can increase contribution densities. Higher contribution densities then can allow workers to afford earlier retirement. At the same time, the matching implies that workers need to save less to finance a given pension at a given retirement age. Thus, contribution densities can decrease while retirement ages increase.

Figure 11: Reform 1 - Minimum Retirement at 55 - No Minimum Pension

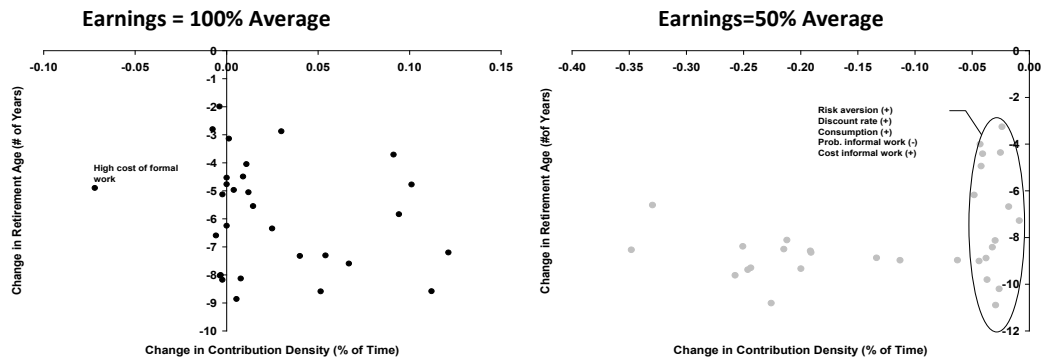


Note: Comparisons are relative to baseline  
Source: Simulation model.

Next we add a minimum pension guarantee offered as a top-up (i.e., 100 percent claw back rate) that is available starting age 55 (Reform 2). We see that for average earners there are no major changes. This is because these workers are less likely to benefit from the minimum pension in the first place. For low income workers, however, the minimum pension reduces substantially retirement ages and contribution densities. Indeed, workers can now contribute less, become eligible for a minimum pension and thus be able to replace a substantial part of their earnings at early ages. The effect of the minimum pension on the retirement age has been previously discussed in Bodor et al. (2008) and Jiménez-Martin and Sánchez-Martin (2006). These new results, in addition, emphasize the negative effect that the minimum pension can have on the time that workers spend in the formal sector. Indeed, simulated reductions in contribution densities are considerable, ranging between 10 and 30 percentage points.



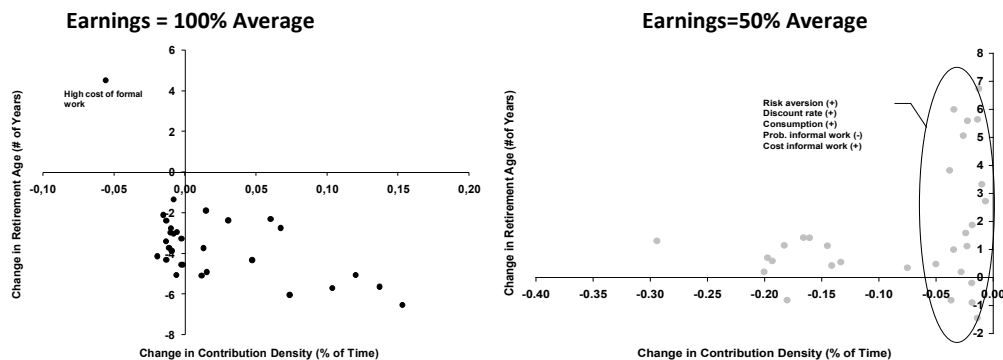
Figure 12: Reform 2 - Minimum Pension at Age 55



Note: Comparisons are relative to baseline  
Source: Simulation model.

The next simulation increases the eligibility age for the minimum pension to 65 (Reform 3). Again, there are no major changes for average earners – except that a few might delay retirement and reduce contribution densities thus becoming eligible for the minimum pension. Among low income workers, on the other hand, the effects are large. First, not surprisingly, the minimum pension at age 65 creates strong incentives to delay retirement and the majority of workers do. Contribution densities also decrease in all cases but most of the time the effect is small – less than 5 percentage points. In fact, there is a very strong negative correlation (-0.9) between the increase in the contribution density and the increase in the retirement age. Those individuals who delay retirement until age 65 leave unchanged or increase little the contribution density (i.e., lower pension wealth from the minimum pension provides some incentives to earn more and save more). On the other hand, those workers who do not increase retirement ages (mainly because retirement ages are already high) “accommodate” the subsidies by reducing contribution densities.

Figure 13: Reform 3 – Minimum Pension at 65



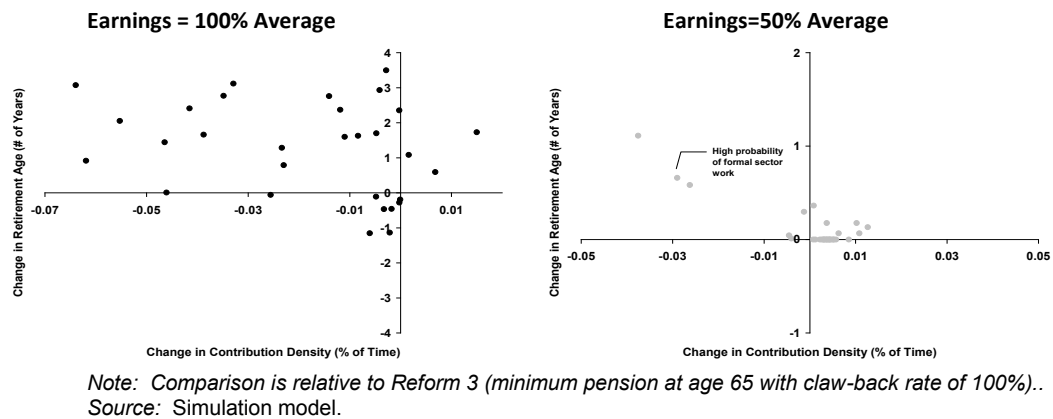
Note: Comparisons are relative to baseline  
Source: Simulation model.

We also look at the effects of the claw back rates which are usually introduced to improve incentives to contribute (Reform 4). As discussed in Section 2, the issue is that in the case of a minimum pension offered as a top-up (100% claw back) each unit increase in the contributory pension is offset by a one unit decrease in the minimum pension guarantee: there is a 100% marginal tax on the contributory pension. This reduces incentives to contribute beyond a minimum necessary to be eligible for the minimum pension. In theory, other things being equal, reducing the claw-back rate can increase contribution densities (see Valdez-Prieto, 2008; Piggot et al. (2009)). This assumes, however, that there are no other costs involved in taking formal sector jobs (or other benefits from informal sector jobs). When this is not the case, the income effect resulting from the reduction in the marginal tax can actually reduce contribution densities. Basically, some individuals would be able to “afford” reducing efforts and spending less time in the formal sector, having lower earnings, and lower savings.

To test these two possible cases we simulate the impact of moving from a 100% claw back rate to a 0% claw back rate. In essence, we are moving from a top-up to a flat pension – where in theory individuals would not mind contributing more since they would not lose the minimum pension. The results show that in the case of average earners contribution densities would actually decrease (although the changes are small, below 5 percentage points) while retirement ages increase (see left bottom panel of **Figure 14**). Basically, individuals have incentives to delay retirement and benefits from the flat pension. At the same time, the increase in pension wealth allows them to reduce efforts to find and keep formal sector jobs. Thus, the correlation between changes in contribution densities and the changes in retirement ages is -0.87.

For low income individuals the situation is different. Despite the transfer, they are still better off by contributing the same or a bit more and increasing the contributory pension -- and the 0% marginal tax provides some incentives to do so. Thus, in most cases, contribution densities increase, albeit by not much (less than 2 percentage points). The few exceptions where contribution densities go down involve individuals with a high probability of finding formal sector jobs. They have higher contribution densities to start with. The income effect from the flat pension then allows them to reduce efforts somewhat and spend less time in formal sector jobs, although the effects are also small (less than 3 percentage points).

Figure 14: Reform 4 - Minimum Pension at Age 65 with Claw Back Rate of 0%



Our next simulations look at the effect of matching contributions (Reforms 5 and 6) which have been suggested as a promising instrument to expand pension coverage to the informal sector (see Palacios and Robalino, 2009). In essence, the program involves matching part of the contributions made by employees (and in this case the employer) as an incentive to promote enrollment and contributions, and thus help finance an adequate pension at retirement for individuals with low savings capacity.

In our example we look at two matching levels: 75% (Reform 5) and 225% (Reform 6). In both cases we allow for two retirement ages, minimum 55 and minimum 65. The analysis is only applied to low income workers. Most high income workers here are not eligible for the minimum pension guarantee and therefore would not be eligible for matching contributions either.

The results are summarized in **Figure 15**. The two panels give the changes in contribution densities and retirement ages for the two matching levels relative to the case of the minimum pension guarantee at age 65 (Reform 3). In the figures the circles correspond to the case with a minimum retirement age at 55 and the squares to the case with a minimum retirement of 65 years. In the second panel we have also included a case where individuals can retire after age 55 but before age 65, as long as they have a pension that is above or equal to the minimum pension (see triangles).

We observe that in all cases the matching increases contribution densities. The most significant effects, however, are seen when there are no restrictions on the retirement age. Contribution densities in that case can increase, on average, by up to 30 percentage points depending on preferences. The tradeoff, however, is a reduction in the retirement age that can decline by up to 10 years. In essence, individuals who before delayed retirement to benefit from the minimum pension offered at age 65 now are able to retire early and benefit from the matching.

This tradeoff had already been discussed above: other things being equal, when individuals delay retirement, they often can afford to contribute less to the pension system and vice versa.

One policy implication would be that, within a strategy to expand coverage and promote formality, matching contributions can play a role as long as individuals are allowed to decide when to retire. As long as the pension system is actuarially fair and there is a maximum level of matching, this would not threaten the financial sustainability of the system. Imposing restrictions on the retirement age and, in particular, setting a high retirement age would reduce incentives to contribute and participate in the social security.

Unfortunately, not imposing a higher minimum retirement age and not having a minimum pension can result in pension values that are too low relative to earnings. The alternative then is to set a retirement restriction that is based on the value of the pension. For instance, individuals could retire at any age below a minimum (e.g., age 65) but only if the pension they receive is equal or above the minimum pension. The triangles in the second panel of **Figure 15** show that under this policy contribution densities increase more than when individuals are simply forced to retire at age 65, although less than when the minimum retirement age is set at 55. At the same time, there are fewer incentives to reduce retirement ages.

Still, even with restrictions in the value of the pension necessary to retire, many individuals are likely to end up with pensions that are too low. This can be seen in **Figure 16** which graphs the average replacement rate received by individuals with different preferences as a function of the costs. The various markers in the figure refer to alternative policies. We see, for instance, that in the case of the three policies that offer a matching of 225% (at age 65, at age 55, and before age 65 if the pension is above the minimum) many workers retire with a pension that represents less than 84 percent of pre-retirement earnings (meaning below the minimum pension which is equal to 42 percent of average earnings). In essence, although workers have higher contribution densities, these are often not enough to finance the current value of the minimum pension. Even in the case of matching contributions with a restriction in the value of the pension before age 65 (see triangles) individuals can end up with pensions below the minimum.

Figure 15: Reforms 5 & 6 – Effects of Matching Contributions

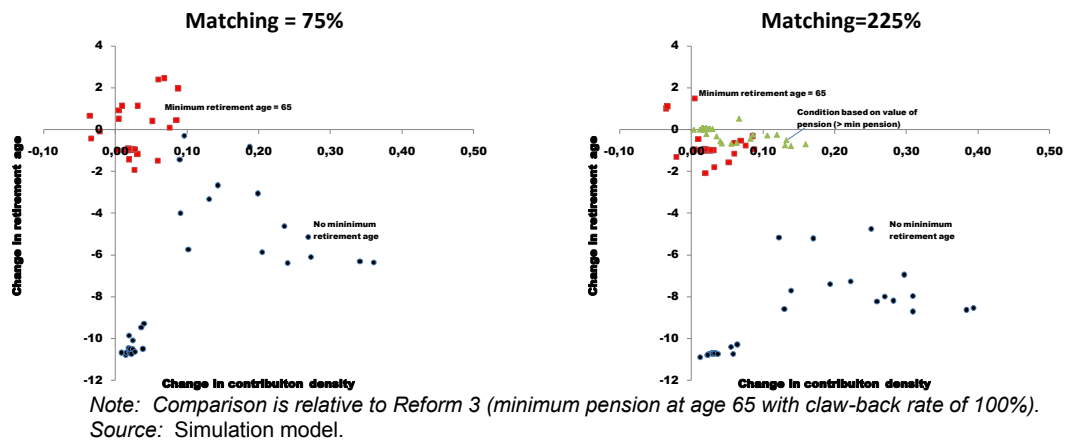
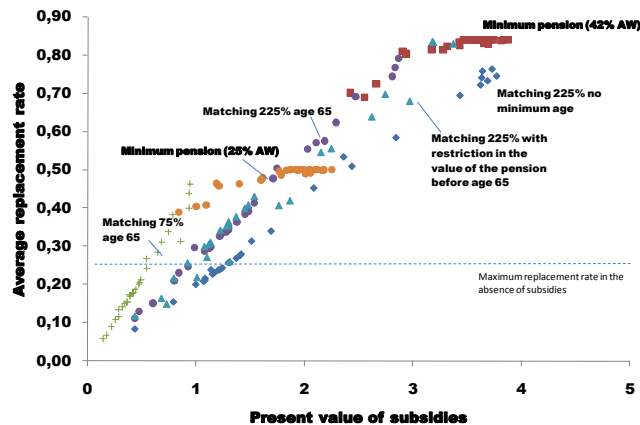


Figure 16: Reforms 5 & 6 – Costs of Matching Contributions



At the same time, in the majority of cases, the cost of matching contributions is lower than the cost of the minimum pension (i.e., for a given level of the replacement rate costs are lower for the matching<sup>15</sup>). Indeed, in the case of the two minimum pensions graphed (42 and 25 percent of average earnings) the majority of workers retire with a pension at least equal to the minimum (there are a few exceptions of workers retiring before age 65 with no minimum pension), but this means that the pension system needs to subsidize a large part of the total pension received. The subsidy is by definition larger than in the case of matching contributions since workers contribute less (i.e., have lower contribution densities).

<sup>15</sup> The exception is the case with matching and no restriction in the minimum age since individuals can receive a given replacement rate at an early age.

It is also important to note that while matching contributions may fail to bring most individuals to retire with, effectively, a replacement rate equal to 84 percent of pre-retirement earnings (or 42 percent of the average wage) it can bring many workers to retire with a replacement rate of 50 percent (or 25 percent of the average). And replacement rates with matching are considerably higher than replacement rates without matching (the maximum replacement rate without matching is represented in the figure by the dotted horizontal line). Hence, the effectiveness of matching contributions also depends on the policy objective. By international standards a minimum pension equal to 42 percent of average earnings is high (see Whitehouse, 2007). A 25 percent target would be more affordable and could be more easily achievable through matching contributions. Or, one could think of a system where the minimum is 25 percent but where individuals who contribute more can still finance higher pensions without losing the subsidies (which is the case that is being illustrated here).

This being said, there is still the problem of those individuals who, despite the incentives, do not contribute enough and end up with very low pensions (e.g., below 25 percent of average earnings). The solution in this case, we argue, would be to offer a transfer to those individuals with a consumption level below a certain minimum – this minimum in fact would apply regardless of age. It would be an anti-poverty means-tested transfer and the test would involve consumption (or total earnings) not simply pension income. Simulations, not presented here, show that a flat transfer equal or below 20 percent of average earnings for those individuals whose consumption fall below that level would not have significant effects on behaviors.

## 6 Conclusions

This paper contributes to the literature both in terms of analytical methods and policy analysis. On methods, we solved and estimated an inter-temporal behavioral model that can be used to analyze how changes in the rules of pensions and unemployment benefits systems affect contribution densities (that is decisions to participate in the social security), savings, retirement ages, and program costs. This type of model can be a complement to the standard non-behavioral models used traditionally in the analysis of pensions and unemployment insurance reforms.

We also developed a Bayesian methodology to estimate the joint distribution of model parameters based on a generalized version of the Gibbs sampler -- the so called Metropolis-Hastings (MH) algorithm. In our application the estimation used as the target the distribution

across four states (contributing to the social security, out of the social security, unemployed, or retired) of a representative cohort of males living in urban areas in Brazil.

In terms of policy analysis our application of the model to Brazil provides several insights about the reform of pensions and income protection systems.

First we find that there are important interactions between the pension system, the unemployment insurance system, and the FGTS (the system of individual unemployment savings accounts). Changes in the unemployment insurance system, for instance, affect contribution densities that in turn affect retirement ages, pension levels and therefore the costs of the pension system. Similarly, changes in FGTS and the pension system affect the performance of the UI system. The main implication is that the design and implementation of reforms across these two programs should be coordinated.

Regarding incentives we find that the current programs affect savings, contribution densities, and retirement ages through complex interactions. Effects are very sensitive to individual preferences and income levels. In general, it is questionable whether the calculation of an “average effect” (say in retirement ages or contribution densities) for a given change in program rules is sufficient to inform policy. Indeed, this average effect would hide considerable variation related to unobserved heterogeneity in preferences. Thus, we have focused on the analysis of policy impacts *across* the distribution of model parameters.

Globally, the results show that the pension system provides incentives to delay retirement for both high and income workers. It also reduces the contribution densities of high income workers and can increase or decrease the contribution density of low income workers. The unemployment insurance system also increases retirement ages, reduces the contribution densities of high income workers and can increase or decrease the contribution densities of low income workers. Finally, FGTS has little effect on the retirement ages of high income workers but increases the retirement ages of low income workers. In both cases it increases contribution densities.

The results also support the idea that financial sustainability and efficiency in the Brazilian social insurance system could improve by making redistribution more explicit and transparent. In the case of pensions this would imply adopting a single actuarially fair formula that links contribution to benefits (without the need to move to a funded scheme) and is “incentives neutral.” Targeted retirement income transfers would then be used to provide incentives to enroll and contribute to low income workers and to top-up their benefits. To this end, matching contributions combined with anti-poverty transfers appear as a better option than the current minimum pension guarantee. Indeed, matching contributions provide better incentives to contribute and are less

costly. A similar reform could be considered for the income protection system. The idea there is to unify the unemployment insurance system and FGTS. The core of the unemployment benefits would come from FGTS (again minimizing distortions), while the UI component (and part of the dismissal tax) would be used to finance explicit redistribution within the system (i.e., to top up the benefits of low income individuals).

There are, of course, limitations to the analysis. First, the model remains a simplified representation of reality. While it can reproduce the distribution of a given cohort across states, there is no guarantee that it is a fair representation of how individuals react to *change* (even if their preferences remain the same). For instance, as Prospect Theory tells us, individuals might react differently to gains than to losses. We have addressed this by looking at a broad range of possible behavioral responses, but still the results should be interpreted with caution.

The second limitation is that we work in a partial-equilibrium framework. Several of the reforms discussed here are likely to affect the demand for labor and equilibrium wages and this would influence the steady-state impact of the proposed reforms.

Third, given the considerable demands on computing time, we have not been able to look at the pensions and unemployment insurance systems in their totality, and have focused instead on a single age-gender cohort and two income levels. The net effect of the reforms and their costs would of course depend on the distribution of individuals by income groups. Moreover, other individual characteristics (e.g., education) are likely to be relevant in determining individual preferences and therefore behavioral change. We also know that our estimates of the posterior joint distribution of model parameters could be improved if working with individual records instead of a pseudo panel. We expect that this will be possible in the future.

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## Annex 1: Benefit Formulas in the Pensions and UI Systems

### Pensions

The proportional Length of Contribution Pension is Given by:

$$p_R = \frac{0.31 * v}{G(R)} \left( 1 + \frac{0.31 * v + R}{100} \right) * LifeTimeWage * 0.7 * \min\left(\frac{1}{0.70}, (1 + 0.05 * (R - 54))\right)$$

where R is the retirement age, v the vesting period, G(R) life expectancy at age R, and *LifeTimeWage* is the average of all salaries indexed by inflation.

The full Length of Contribution Pension is given by:

$$p_R = \frac{0.31 * v}{G(R)} \left( 1 + \frac{0.31 * v + R}{100} \right) * LifeTimeWage$$

The aging pension is:

$$p_R = (0.7 + 0.01 * v) * LifeTimeWage$$

### Unemployment Insurance

The value of monthly UI benefits varies from R\$380 (the Brazilian Minimum Wage) to R\$710.97, depending on the average wage computed in the last three-month period of work. Values are depicted in the table below.

Monthly wage range	UI benefit
Up to R\$627.29	0.8 * average monthly wage Minimum value = 1 Minimum Wage (R\$380.00)
R\$627.30 to R\$1,045.58	R\$501,83 + 0.5 * value exceeding R\$627.29 Maximum value = R\$710.97
More than R\$1,045.58	R\$710.97

## Annex 2: Moving from Cross-Sectional to Longitudinal Cohorts

The main source of information used in this paper are the microdata from the Pesquisa Nacional por Amostra de Domicílios (PNAD) –National Household Sample Survey. This survey is goes to the field each year (except the years of the Census) and is managed by the Instituto Brasileiro de Geografia e Estatística (IBGE) - Brazilian Institute of Geography and Statistics. It is a comprehensive research on socio-economic characteristics of the population and households in Brazil. The issues include topics such as income, occupations, social security, education, fertility, etc. Each year are interviewed around 0.25% of the Brazilian population, which corresponds to just over 420,000 records. For this exercise we used the PNADs for years 1990, 1996, 2001, and 2006. In addition, we relied on aggregate data from the Statistical Yearbook of Social Security, published yearly by the Ministry of Social Security.

For the analysis the population was divided in the following groups:

- a) *Workers "with carteira."* This group includes all individuals that work in the formal private sector. Or, for the purposes of this paper, which contribute to the social security system. Thus, civil servants and military are excluded from our analysis
- b) *Workers "without carteira."* This group includes all workers who are in the informal sector. Or, for the purposes of this paper, which do not contribute to the basic social security system.
- c) *Unemployed.* This group included all unemployed individuals. It means they are not working they are looking for a job.
- d) *Retired workers.* This group includes all who receive old-age benefits and are not in the labor market. This caveat is important, because in Brazil a worker can retire, receive their benefit and continue working, without any changes, either in his situation in the labor market, either in his situation as beneficiary.

People were then divided by cohorts of 5 years, according to the following division: 16-20 years; 21-25 years;...; 66-70; and 71+. All persons under the age of 16 years were excluded from the dataset because that is the legal age of initiation of work and contribution to social security.

The main complication at this stage was with the retirees. The PNAD does not provide information on whether the person receives his retirement from the Regime Geral de Previdência Social (RGPS) - General Social Security System or the National Social Security Institute (i.e., if the person is a retiree from the private or public sector). To this end, the data from the survey was matched to the data from the Anuário Estatístico de Previdência Social (AEPS) - Statistical Yearbook of Social Security, which contains retirees by sector and age groups. Given that the age groups are not the same some additional adjustments were necessary. As the age-cohorts made by AEPS start from an age  $x$  and our cohorts from an age  $x + 1$ , the two information have only 4 years in common (80% of data). Thus, we built a new cohort  $x'$ , composed of  $0.8 * (\text{similar cohort of AEPS}) + 0.2 * (\text{previous cohort of AEPS})$ .

### Moving from cross-sectional to longitudinal cohorts

With the pseudo-panel formed by the PNADs it is possible to describe the behavior along the life cycle of a few cohorts. For example, the cohort aged  $x$  in 2006 was  $x-5$  years old in 2001,  $x-10$  years old in 1996 and so on. But the question we want to answer is: what will happen with that cohort in 5 years from now? And in  $n$  years from now? This means trying to predict the percentage of individuals of that cohort that will be in each of the four groups that defined above in the next  $n$  years. We considered two methods.

**Method 1.** The assumption here is that the behavior of a given cohort will be similar to what happened with individuals of other cohorts, (that can be observed in other PNADs), when they were the same age. For example, in 2006 a given percentage of individuals aged  $x$  were in the formal sector. In 2011, this cohort will be 5 years older. We postulate that the percentage of individuals from the cohort who would be in the informal sector is a weighted average of percentages found for the cohort aged  $x + n$  in the previous PNADs. The weighting gives greater importance to more recent years. For each cohort and for each group, the procedure is repeated. So we have:

$$PCT_{x,i,j} = \frac{a * PCT_{x,i,j-5} + b * PCT_{x,i,j-10} + c * PCT_{x,i,j-15} + d * PCT_{x,i,j-21}}{a + b + c + d}$$

where  $PCT_{xij}$  is the percentage of people who was in the group  $i$ , aged  $x$ , in the year  $j$ . The terms  $a$ ,  $b$ ,  $c$  and  $d$  represent the weights of each year in the equation.

**Method 2.** In this case, for each cohort, we estimate the relationship between the percentages of individuals in each group found in each pair of consecutive PNADs. For instance, in the first PNAD there is a given percentage for people aged  $x$  that is in category  $y$ . In the following PNAD, we take into account the age group  $x + n$  (i.e., the same cohort) and look at the percentage still in category  $y$ . We compute the growth rate  $RG$ :

$$RG = 1 - \frac{PCT_{x,i,j}}{PCT_{x-5,i,j-5}}$$

After this, we calculated the mean rates for going from age  $x$  to age  $x+5$  for every category. We used these means to input values for the distribution of the cohorts in the years when we do not observe them. The results for each group are normalized in such a way that the sum of the four groups is always 100.

Methods 1 and 2 give similar results for workers with *carteira* (see Table). For workers without *carteira* the second method seems to overestimate this group, especially for older workers. For the unemployed and the retired the same thing occurs, the difference is more important for the latter. In the analysis we therefore opted for the first method.

Age (2006)	Method 1	Method 2	Longitudinal data
31-35	52,56	52,56	52,56
36-40	52,54	49,53	51,49
41-45	51,64	45,28	51,66
46-50	48,90	39,04	49,44
51-55	41,86	30,47	43,10
56-60	30,46	21,19	30,93
61-65	17,09	12,35	16,33
66-70	4,80	4,29	4,35
71+	0,93	0,95	0,87

Source. Authors' calculations.

### Annex 3: First Order Expansions of the Behavioral Model

#### Average Earnings (Contribution Density)

Name variable	Code	Coeff	StErr	Included in final model?	Partial Derivative
<i>Risk Aversion</i>	ra	-8,371	1,236	Yes	0,02
<i>Time Pref</i>	tp	-36,505	14,175	Yes	-0,06
<i>Consumption Pref</i>	cp	0,280	0,561	No	0,03
<i>Prob Formal Work</i>	pfw	-7,325	3,431	Yes	0,16
<i>Desutility effort</i>	de	-10,495	2,372	Yes	-0,19
<i>Prob working if Retired</i>	pwr	0,875	0,891	No	-0,01
	ra^2	4,767	0,592	Yes	
	tp^2	-12,431	38,942	No	
	cp^2	0,094	0,367	No	
	pfw^2	8,481	2,094	Yes	
	de^2	12,876	3,101	Yes	
	pwr^2	-0,743	0,375	Yes	
	ra*tp	-9,774	8,615	No	
	ra*cp	0,137	0,449	No	
	ra*pfw	0,304	2,151	No	
	ra*de	-7,853	1,424	Yes	
	ra*pwr	0,916	0,974	No	
	tp*cp	57,347	12,499	Yes	
	tp*pfw	-38,562	14,188	Yes	
	tp*de	7,103	17,975	No	
	tp*pwr	-7,784	9,641	No	
	cp*pfw	0,292	0,625	No	
	cp*de	9,166	2,609	Yes	
	cp*pwr	0,839	1,100	No	
	pfw*de	-4,198	3,481	No	
	pfw*pwr	0,533	0,962	No	
	de*pwr	-0,035	3,177	No	

Source. Authors' calculations.

### 50% Average Earnings (Contribution Density)

Variable Name	Code	Coeff	StErr	Included in final model?	Partial Derivative
<i>Risk Aversion</i>	ra	-11,683	1,659	Yes	0,05
<i>Time Pref</i>	tp	-126,951	19,251	Yes	-0,05
<i>Consumption Pref</i>	cp	47,129	10,349	Yes	0,06
<i>Prob Formal Work</i>	pfw	10,385	4,124	Yes	0,13
<i>Desutility effort</i>	de	-10,130	2,110	Yes	-0,12
<i>Prob working if Retired</i>	pwr	-2,871	5,169	No	-0,02
	ra^2	5,126	0,953	Yes	
	tp^2	-92,245	36,863	Yes	
	cp^2	-13,566	5,533	Yes	
	pfw^2	7,542	1,851	Yes	
	de^2	9,799	5,033	Yes	
	pwr^2	1,053	3,241	No	
	ra*tp	-1,408	14,314	No	
	ra*cp	-3,653	3,555	No	
	ra*pfw	1,137	1,379	No	
	ra*de	-1,446	2,944	No	
	ra*pwr	6,512	2,450	Yes	
	tp*cp	154,610	25,688	Yes	
	tp*pfw	11,149	14,747	No	
	tp*de	-62,848	20,094	Yes	
	tp*pwr	2,886	17,494	No	
	cp*pfw	-24,980	4,389	Yes	
	cp*de	1,106	6,341	No	
	cp*pwr	-17,995	4,093	Yes	
	pfw*de	2,212	3,124	No	
	pfw*pwr	6,727	1,913	Yes	
	de*pwr	-2,376	5,013	No	

Source. Authors' calculations.

## Average Earnings (Retirement Age)

Variable Name	Code	Coeff	StErr	Included in final model?	Partial Derivative
<i>Risk Aversion</i>	ra	-32.632	10.556	Yes	0.49
<i>Time Pref</i>	tp	-138.140	75.649	Yes	-0.05
<i>Consumption Pref</i>	cp	-69.417	15.568	Yes	0.78
<i>Prob Formal Work</i>	pfw	16.286	11.963	No	-0.59
<i>Desutility effort</i>	de	-108.627	19.705	Yes	-0.58
<i>Prob working if Retired</i>	pwr	10.853	15.447	No	-1.16
	ra^2	27.826	4.712	Yes	
	tp^2	-505.482	240.010	Yes	
	cp^2	27.056	11.507	Yes	
	pfw^2	5.299	9.047	No	
	de^2	-29.803	16.708	Yes	
	pwr^2	8.654	8.759	No	
	ra*tp	-92.287	55.214	Yes	
	ra*cp	25.915	8.129	Yes	
	ra*pfw	-39.321	4.502	Yes	
	ra*de	30.699	15.041	Yes	
	ra*pwr	-49.109	7.883	Yes	
	tp*cp	-40.377	92.944	No	
	tp*pfw	267.561	59.387	Yes	
	tp*de	-340.777	99.349	Yes	
	tp*pwr	175.527	48.989	Yes	
	cp*pfw	-2.054	16.712	No	
	cp*de	40.685	18.543	Yes	
	cp*pwr	9.091	11.980	No	
	pfw*de	91.222	12.322	Yes	
	pfw*pwr	70.820	11.214	Yes	
	de*pwr	-90.023	15.164	Yes	

Source. Authors' calculations.



## 50% Average Earnings (Retirement Age)

Variable name	Code	Coeff	StErr	Included in final model?	Partial Derivative
<i>Risk Aversion</i>	ra	-1.436	4.586	No	0.25
<i>Time Pref</i>	tp	-9.669	32.673	No	-0.10
<i>Consumption Pref</i>	cp	56.988	26.356	Yes	1.95
<i>Prob Formal Work</i>	pfw	8.744	11.708	No	-0.06
<i>Desutility effort</i>	de	-68.608	18.068	Yes	-0.04
<i>Prob working if Retired</i>	pwr	-140.446	20.979	Yes	-2.08
	ra^2	-0.206	2.893	No	
	tp^2	5.557	118.800	No	
	cp^2	-74.833	22.727	Yes	
	pfw^2	-6.770	3.570	Yes	
	de^2	-9.149	22.983	No	
	pwr^2	-130.697	16.001	Yes	
	ra*tp	1.926	25.382	No	
	ra*cp	-1.873	5.764	No	
	ra*pfw	-16.222	4.981	Yes	
	ra*de	30.072	10.201	Yes	
	ra*pwr	24.488	9.936	Yes	
	tp*cp	4.175	44.604	No	
	tp*pfw	105.327	21.437	Yes	
	tp*de	-465.807	92.913	Yes	
	tp*pwr	-43.854	78.415	No	
	cp*pfw	8.183	11.388	No	
	cp*de	-1.230	29.682	No	
	cp*pwr	201.251	26.392	Yes	
	pfw*de	46.814	13.430	Yes	
	pfw*pwr	52.609	10.637	Yes	
	de*pwr	-29.041	25.890	No	

Source. Authors' calculations.